Given:

Lame' Equations for internal pressure

$$\sigma_r = \frac{a^2 p_i}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right) \qquad \sigma_z = \frac{p_i a^2}{b^2 - a^2}$$

$$\sigma_\theta = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

Find: Show that the Lame' Equations reduce to

$$\sigma_\theta = \frac{p_i a}{t}$$

for $\frac{b}{a} \to 1$.

Solution: Let $b = a + t$ where $t \ll a$ or b.

$$\sigma_\theta = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

$$= \frac{a^2 p_i}{\cancel{a^2} + 2at + t^2 - \cancel{a^2}}\left(1 + \frac{a^2 + 2at + t^2}{r^2}\right)$$

$$\simeq \frac{a p_i}{2t}\left(1 + \frac{a^2}{r^2}\right)$$

$$\simeq \frac{a p_i}{2t}(2) = \boxed{\frac{p_i a}{t}}$$

$$\sigma_r = \frac{a^2 p_i}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

$$\simeq \frac{a p_i}{2t}\left(1 - \frac{a^2}{r^2}\right)$$

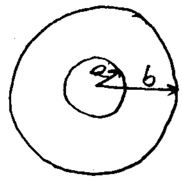
$$\simeq 0$$

$$\sigma_3 = \frac{p_i a^2}{b^2 - a^2} = \frac{p_i a^2}{(a+t)^2 - a^2} = \frac{p_i a^2}{\cancel{a^2} + 2at + \cancel{t^2} - \cancel{a^2}}$$

$$= \frac{p_i a^2}{2at} = \frac{p_i a}{2t}$$

$$\sigma_3 \simeq \frac{p_i a}{2t}$$

Given:



$p_i = 10\,ksi$

$a = 6$ inch

$b = 8$ inch

$S_{yt} = 36\,ksi$

$\nu = 0.3$

Closed but unconstrained ends.

Find: Von Mises stress

Solution:

The maximum stresses will be at $r = a$.

$$\sigma_r = \frac{a^2 p_i}{b^2 - a^2}\left(1 - \frac{b^2}{a^2}\right)$$

$$= \underline{\underline{-10\,ksi}}$$ (Note that this satisfies the pressure boundary condition)

$$\sigma_\theta = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right)$$

$$= \underline{\underline{35.7\,ksi}}$$

$$\sigma_z = \frac{a^2 p_i}{b^2 - a^2} = \underline{\underline{12.9\,ksi}}$$ (Assumed to be closed but unrestrained)

$$\sigma_{eff} = \sqrt{\sigma_r^2 + \sigma_\theta^2 + \sigma_z^2 - \sigma_r\sigma_\theta - \sigma_\theta\sigma_z - \sigma_z\sigma_r}$$

$$= \underline{\underline{39.6\,ksi}}$$ Material has yielded at inner surface