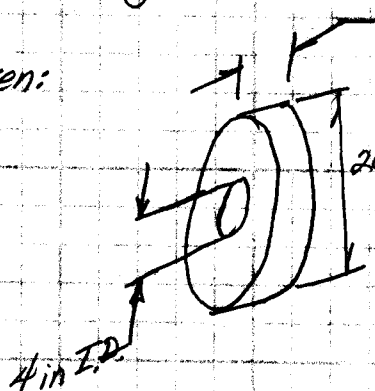
Given:



Rotational Speed = 6900 RPM

$\rho = 0.00072 \frac{lbf\text{-}sec^2}{in^4}$

$E = 30 \times 10^6 psi$

$\nu = 0.3$

Find: (a) Required interference (inches);
(b) maximum stress when not rotating; and
(c) maximum stress when rotating.

Solution: The radial displacement of the shaft (a) at its o.d. is given by

$$u_s = \frac{1-\nu}{8E}\left[(3+\nu)b^2 - (1+\nu)b^2\right]\rho b\omega^2$$

$$= \frac{1-\nu}{8E}(2b^2)(\rho b\omega^2) \qquad \omega = \left(6900\frac{rev}{min}\right)\left(\frac{2\pi rad}{rev}\right)\left(\frac{min}{60sec}\right)$$

$$= \frac{1-\nu}{8E}(\rho\omega^2)(2b^3) \qquad = 722.6\ rad/sec$$

$$= \frac{(1-0.3)\left(0.00072\frac{lb\text{-}sec^2}{in^4}\right)\left(722.6\frac{rad}{sec}\right)^2(2)(2in)^3}{8\left(30\times10^6\ lb/in^2\right)}$$

$$= 1.75 \times 10^{-5}\ in$$

the radial displacement of the hub at its i.d. is given by

$$u = \frac{(3+\nu)(1-\nu)}{8E}\left(a^2 + b^2 - \frac{1+\nu}{3+\nu}a^2 + \frac{1+\nu}{1-\nu}b^2\right)\rho a \omega^2$$

$b = 10\,in$ $\quad \omega = 722.6\,rad/sec$
$a = 2\,in$

$$\Rightarrow u_h = \frac{(3.3)(0.7)\left(4 + 100 - \frac{1.3}{3.3}(4) + \frac{1.3}{0.7}(100)\right)}{8\,(30\times10^6\,lb/in^2)}$$

$$* \left(0.00072\,\frac{lb\text{-}sec^2}{in^4}\right)(2in)\left(722.6\,\frac{rad}{sec}\right)^2$$

$$= 2.085\times10^{-3}\,in$$

The minimum interference is the difference between u_h & u_s

$$\delta = u_h - u_s = 2.085\times10^{-3} - 1.75\times10^{-5} = \underline{\underline{2.07\times10^{-3}\,in}}$$

(b) When not rotating, the interference pressure is given by

$$p = \frac{\delta}{a\left[\frac{b^2+a^2}{b^2-a^2} + \nu_h + \frac{1-\nu_s}{E_s}\right]}$$

Reference Lecture 15

$$p = \frac{2.07\times10^{-3}\ in}{2in\left[\frac{10^2+2^2}{10^2-2^2}+0.3}{30\times10^6\ psi} + \frac{1-0.3}{30\times10^6\ psi}\right]} = 14.9\ ksi$$

The maximum stress will be in the hub,

$$\sigma_\theta|_a = p\frac{a^2+b^2}{b^2-a^2} = 14.9ksi\left(\frac{2^2+10^2}{10^2-2^2}\right) = 16.1\ ksi$$

Reference Lecture 14

(c) When rotating, the maximum hoop stress in the hub is given by

$$\sigma_\theta|_a = \frac{3+\nu}{8}\left(a^2+b^2-\frac{1+3\nu}{3+\nu}a^2+b^2\right)\rho w^2$$

Reference Lecture 16

$$= \frac{3+0.3}{8}\left(2^2+10^2-\frac{1+3(0.3)}{3+0.3}2^2+10^2\right)(0.00072)(722.6)^2$$

$$= 31.3\ ksi$$