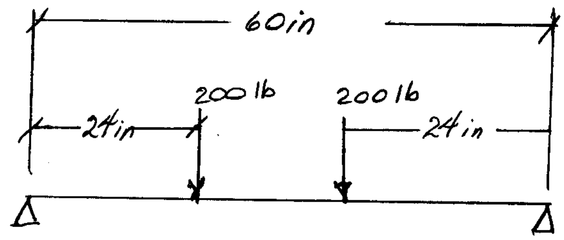
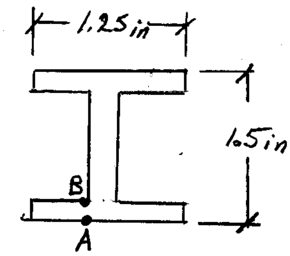
Given:





$S_y = 50$ ksi

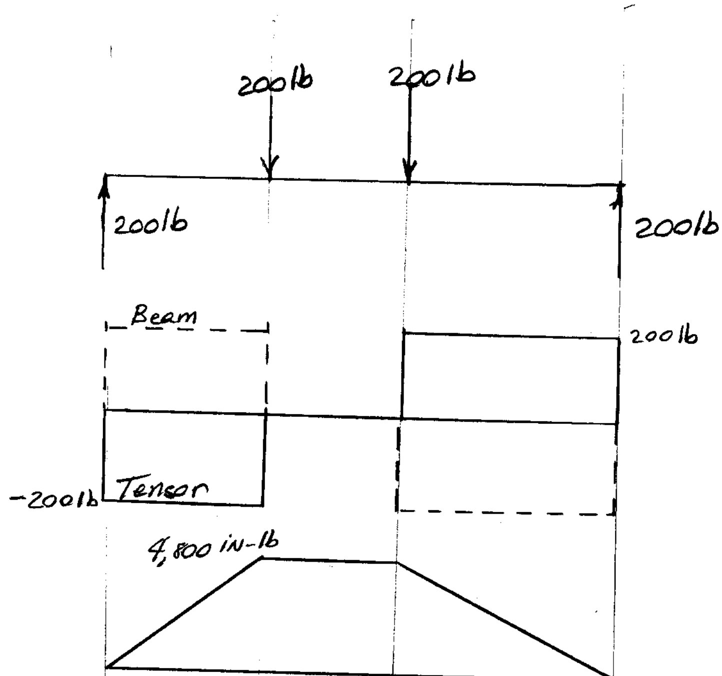
Web and flange thickness = 0.125 in

Find: (a) Bending and transverse shear stress at A & B.

(b) Maximum normal and maximum shear stress.

(c) Factor of safety based on maximum normal and maximum shear stress.

Solution: Find maximum shear force and bending moment.



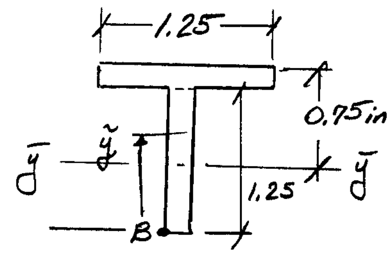
Note that tensor sign convention is used.

$V_{max} = \pm 200$ lb

$M_{max} = 4{,}800$ in-lb

Compute cross section properties

Find Q_B



$$A = (1.25\,in)(0.125\,in) + (1.25\,in)(0.125\,in) = 0.3125\,in^2$$

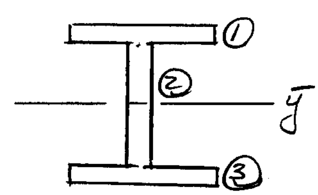
$$\tilde{y} = \frac{(1.25\,in)(0.125\,in)\left(1.25 + \frac{0.125}{2}\right) + (1.25)(0.125)\left(\frac{1.25}{2}\right)}{0.3125}$$

$$= \frac{0.302}{0.3125} = 0.969\,in$$

$$\tilde{y} - \bar{y} = 0.969\,in - 0.625 = 0.344\,in$$

$$Q_B = A(\tilde{y} - \bar{y}) = \underline{\underline{0.1075\,in^3}}$$

Find I



$$I = \sum \bar{I} + (\tilde{y} - \bar{y})^2 A$$

$$= 2\left(\tfrac{1}{12}(1.25\,in)(0.125\,in)^3 + (1.25\,in)(0.125)(0.75 - 0.125/2)^2\right)$$

$$+ \tfrac{1}{12}(0.125\,in)(1.25\,in)^3$$

$$= \underline{\underline{0.168\,in^4}}$$

Part A

Bending stress at A $\quad \sigma_A = \dfrac{Mc_A}{I} = \dfrac{(4{,}800 \text{ in-lb})(0.75 \text{ in})}{0.168 \text{ in}^4}$

$$= \underline{\underline{21.4 \text{ ksi}}}$$

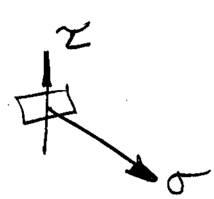
Bending stress at B $\quad \sigma_B = \dfrac{Mc_B}{I} = \dfrac{(4{,}800 \text{ in-lb})(0.625 \text{ in})}{0.168 \text{ in}^4}$

$$= \underline{\underline{17.9 \text{ ksi}}}$$

Transverse shear stress at A $\quad \tau = \dfrac{V\varphi_A}{It} = 0 \quad$ since $\varphi_A = 0$

Transverse shear stress at B $\quad \tau = \dfrac{V\varphi_B}{It} = \dfrac{(200 \text{ lb})(0.1075 \text{ in}^3)}{(0.168 \text{ in}^4)(0.125 \text{ in})}$

$$= \underline{\underline{1.02 \text{ ksi}}}$$



PART B

At point A the principal stresses are

$$\sigma_1 = \sigma = 21.4 \text{ ksi}, \quad \sigma_2 = \sigma_3 = 0 \qquad \underline{\underline{\sigma_{max} = 21.4 \text{ ksi}}}$$

At point B,

$$\sigma_1 = \frac{\sigma_x + \cancel{\sigma_y}}{2} + \sqrt{\left(\frac{\sigma_x - \cancel{\sigma_y}}{2}\right)^2 + \tau_{xy}^2} = 17.95 \simeq 18.0 \text{ ksi}$$

$$\sigma_2 = \frac{\sigma_x + \cancel{\sigma_y}}{2} - \sqrt{\left(\frac{\sigma_x - \cancel{\sigma_y}}{2}\right)^2 + \tau_{xy}^2} = -0.06 \text{ ksi} \qquad \underline{\underline{\sigma_{max} = 18.0 \text{ ksi}}}$$

$$\sigma_3 = 0$$

Maximum shear stress at A $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{21.4}{2} = \underline{\underline{10.7\ ksi}}$

at B $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{18-(-0.06)}{2}$

$= 9.03\ ksi$

PART C

Maximum normal stress theory (Point A)

$$\frac{\sigma_{max}}{S_y} = \frac{1}{N} \Rightarrow N = \frac{S_y}{\sigma_{max}} = \frac{50\ ksi}{21.4\ ksi} = \underline{\underline{2.34}}$$

Maximum shear stress theory (Point A)

$$\frac{\sigma_1 - \sigma_3}{S_y} = \frac{1}{N} \Rightarrow N = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{50\ ksi}{21.4\ ksi} = \underline{\underline{2.34}}$$

Maximum normal stress theory (Point B)

$$\frac{\sigma_{max}}{S_y} = \frac{1}{N} \Rightarrow N = \frac{S_y}{\sigma_{max}} = \frac{50\ ksi}{18.0\ ksi} = \underline{\underline{2.78}}$$

Maximum shear stress theory (Point B)

$$N = \frac{S_y}{\sigma_1 - \sigma_2} = \frac{50\ ksi}{18.06} = \underline{\underline{2.77}}$$