Given: $\sigma_{eff} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1}$

and

$$\sigma_{eff} = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}}$$

Find: Show that the two equations are equal.

Solution:

$$\sigma_{eff}^2 = \frac{1}{2}\left((\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right)$$

$$= \frac{1}{2}(\sigma_1^2 - 2\sigma_1\sigma_2 + \sigma_2^2$$

$$+ \sigma_2^2 - 2\sigma_2\sigma_3 + \sigma_3^2$$

$$+ \sigma_3^2 - 2\sigma_3\sigma_1 + \sigma_1^2)$$

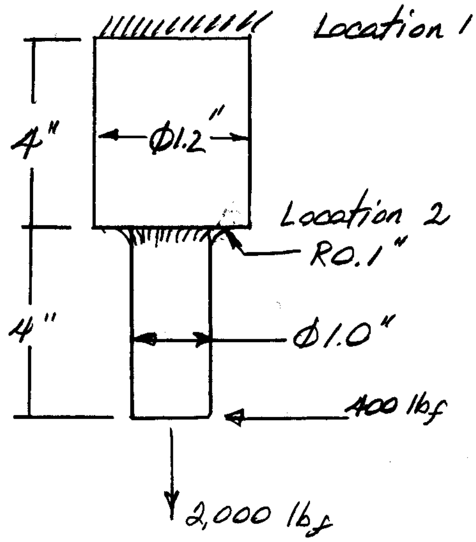
$$= \frac{1}{\cancel{2}}(\cancel{2}\sigma_1^2 + \cancel{2}\sigma_2^2 + \cancel{2}\sigma_3^2$$

$$- \cancel{2}\sigma_1\sigma_2 - \cancel{2}\sigma_2\sigma_3 - \cancel{2}\sigma_3\sigma_1)$$

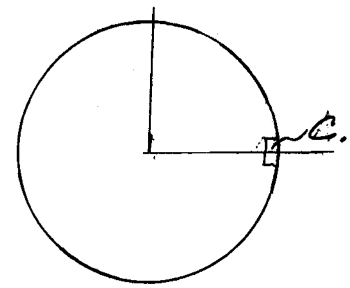
$$\Rightarrow \sigma_{eff} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1}$$

Q.E.D.

Given:



The critical location is a location 2 at point C.



$S_{yt} = 30$ ksi

$\sigma_n = 32.1$ ksi

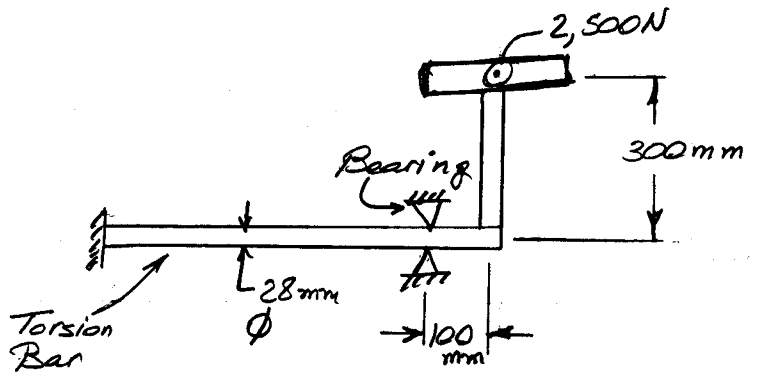
Find: Effective stress and determine if material will yield.

Solution: At location C, there is only a normal force component.

$$\Rightarrow \sigma_{eff} = \sigma_n = 32.1 \text{ ksi}$$

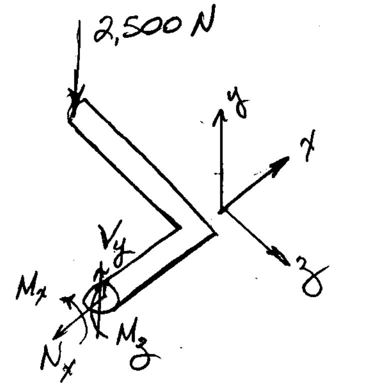
$\sigma_n > S_{yt} \Rightarrow$ material will yield at C.

Given:



Find: Von Mises stress in the torsion bar at the bearing.

Solution



$$\Sigma F_y = 0 = -2,500\,N + V_y \Rightarrow V_y = 2,500\,N$$

$$\Sigma F_x = 0 = N_x \qquad \Rightarrow N_x = 0$$

$$\Sigma M_x = 0 = -(2,500)(0.3\,m) - M_x \Rightarrow M_x = -750\,N\cdot m$$
$$= 750\,N\cdot m$$

$$\Sigma M_z = 0 = -(2,500)(0.1\,m) - M_z \Rightarrow M_z = -250\,N\cdot m$$
$$= 250\,N\cdot m$$

Torsion Stress

$$\tau_{zx} = \frac{M_x\,c}{J}$$

$$c = \frac{0.028\,m}{2} = 0.014\,m$$

$$J = 2I = \frac{\pi}{32}D^4$$
$$= \frac{\pi}{32}(0.028)^4$$
$$= 6.03\times10^{-8}\,m^4$$

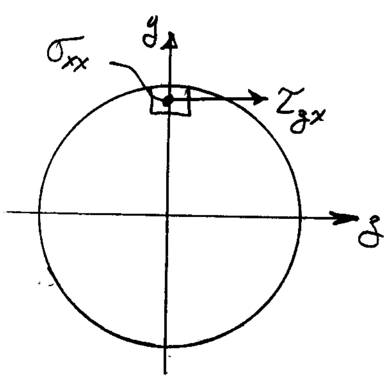
$$\tau_{zx} = \frac{(750\ N \cdot m)(0.014 m)}{6.03 \times 10^{-8} m^4} = 174.\ MPa$$

<u>Bending Stress</u>

$$\sigma_{xx} = \frac{M_z c}{I} \qquad c = 0.014\ m$$

$$I = \frac{J}{2} = 3.02 \times 10^{-8} m^4$$

$$\sigma_{xx} = \frac{(250\ N \cdot m)(0.014 m)}{3.02 \times 10^{-8} m^4} = 116.\ MPa$$



Cross-Section at Bearing

σ_{xx} is coming out of the page.

From Eq. 2-8 in Shigley,

$$\sigma_1 = \frac{\sigma_{xx} + \sigma_{zz}}{2} + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{zz}}{2}\right)^2 + \tau_{zx}^2}$$

$$= \frac{116}{2} + \sqrt{\left(\frac{116}{2}\right)^2 + (174)^2}$$

$$= \underline{\underline{241.\ MPa}}$$

$$\sigma_2 = \frac{\sigma_{xx} + \sigma_{zz}}{2} - \sqrt{\left(\frac{\sigma_{xx} - \sigma_{zz}}{2}\right)^2 + \tau_{zx}^2}$$

$$= \frac{116}{2} - \sqrt{\left(\frac{116}{2}\right)^2 + (174)^2}$$

$$= \underline{\underline{-125.\ MPA}}$$

Writting the principal stresses in ascending order gives

$$\sigma_1 = 241.\ MPa,\ \sigma_2 = 0.,\ \sigma_3 = -125\ MPa.$$

$$\sigma_{eff} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1}$$

$$= \sqrt{(241)^2 + \cancel{(0)^2} + (-125)^2 - \cancel{(241)(0)} - \cancel{(0)(-125)} - (-125)(241)}$$

$$= 322.\ MPa$$

$$\underline{\underline{\sigma_{eff} = 322.\ MPa}}$$