Given:
A hot-rolled bar has a minimum yield strength in tension and compression of 44 ksi.

Find: Factors of safety for MSST and DET failure theories for the following stress states.

(a) $\sigma_x = 9\text{ksi}$, $\sigma_y = -5\text{ksi}$

(b) $\sigma_x = 12\text{ksi}$, $\tau_{xy} = 3\text{ ksi ccw}$

(c) $\sigma_x = -4\text{ksi}$, $\sigma_y = -9\text{ ksi}$, $\tau_{xy} = 5\text{ ksi cw}$

(d) $\sigma_x = 11\text{ksi}$, $\sigma_y = 4\text{ksi}$, $\tau_{xy} = 1\text{ ksi cw}$

Solution: (a) Assume plane stress $\Rightarrow \sigma_z = 0.0$

$$\sigma_1 = 9\text{ksi};\ \sigma_2 = 0.\text{ ksi},\ \sigma_3 = -5\text{ksi}$$

MSST

$$|\sigma_1 - \sigma_3| = \frac{S_{yp}}{N} \Rightarrow N = \frac{S_{yp}}{|\sigma_1 - \sigma_3|}$$

$$= \frac{44\text{ ksi}}{|9-(-5)|}$$

$$N = \underline{\underline{3.14}}$$

DET

$$\sigma_{eff} = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}}$$

$$= \left[\frac{(9)^2+(5)^2+(14)^2}{2}\right]^{1/2} = 12.3\text{ ksi}$$

$$\sigma_{eff} = \frac{S_{yp}}{N} \Rightarrow N = \frac{S_{yp}}{\sigma_{eff}} = \frac{44\text{ ksi}}{12.3\text{ ksi}} = \underline{\underline{3.58}}$$

(b) $\sigma_x = 12 ksi$, $\tau_{xy} = 3$ ksi ccw

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{12}{2} + \sqrt{\left(\frac{12}{2}\right)^2 + (3)^2} = 12.7 \text{ ksi}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{12}{2} - \sqrt{\left(\frac{12}{2}\right)^2 + (3)^2}$$

$$= -0.708 \text{ ksi}$$

Assume plane stress conditions, and order principal stresses highest to lowest.

$\Rightarrow \sigma = 12.7$ ksi, $\sigma_2 = 0.0$, $\sigma_3 = -0.708$ ksi

MSST $\Rightarrow |\sigma_1 - \sigma_3| = \frac{S_{yp}}{N}$

$$\Rightarrow N = \frac{S_{yp}}{|\sigma_1 - \sigma_3|} = \frac{44 \text{ ksi}}{|12.7 - (-0.708)|}$$

$N = \underline{3.28}$

DET $\Rightarrow \sigma_{eff} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}}$

$$= \left[\frac{1}{2}\left[(12.7)^2 + (0.708)^2 + (12.7 - (-0.708))^2\right]\right]^{1/2}$$

$= 13.1$ ksi

$$\sigma_{eff} = \frac{S_{yp}}{N} \Rightarrow N = \frac{S_{yp}}{\sigma_{eff}} = \frac{44 \text{ ksi}}{13.1 \text{ ksi}} = \underline{\underline{3.36}}$$

(c) $\sigma_x = -4\ ksi$, $\sigma_y = -9\ ksi$, $\tau_{xy} = 5\ ksi$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-4-9}{2} + \sqrt{\left(\frac{-4+9}{2}\right)^2 + 5^2}$$

$$= -0.910\ ksi$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-4-9}{2} - \sqrt{\left(\frac{-4+9}{2}\right)^2 + 5^2}$$

$$= -12.1\ ksi$$

Assumming plane stress conditions and ordering the stresses largest to smallest yields

$\sigma_1 = 0.$, $\sigma_2 = -0.910$, $\sigma_3 = -12.1\ ksi$

MSST $\quad |\sigma_1 - \sigma_3| = \frac{S_{yp}}{N} \quad N = \frac{S_{yp}}{|0-12.1|} = \frac{44\ ksi}{12.1\ ksi}$

$$= \underline{\underline{3.64}}$$

$$\sigma_{eff} = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2}{2}}$$

$$= \sqrt{\frac{(0.910)^2 + (-0.910+12.1)^2 + (12.1)^2}{2}} = 11.7\ ksi$$

$$\sigma_{eff} = \frac{S_{yp}}{N} \Rightarrow N = \frac{44\ ksi}{11.7\ ksi} = \underline{\underline{3.76}}$$

(d) $\sigma_x = 11\,ksi$, $\sigma_y = 4\,ksi$, $\tau_{xy} = 1\,ksi$

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{11+4}{2}\right) + \sqrt{\left(\frac{11-4}{2}\right)^2 + 1^2}$$

$$= 11.1\,ksi$$

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 3.86\,ksi$$

Assume plane stress conditions,

$\Rightarrow \sigma_1 = 11.1\,ksi$; $\sigma_2 = 3.86\,ksi$, $\sigma_3 = 0$

MSST $$|\sigma_1 - \sigma_3| = \frac{S_{yp}}{N} \Rightarrow N = \frac{S_{yp}}{|\sigma_1 - \sigma_3|} = \frac{44\,ksi}{|11.1|}$$

$$N = \underline{\underline{3.96}}$$

DET $$\sigma_{eff} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}}$$

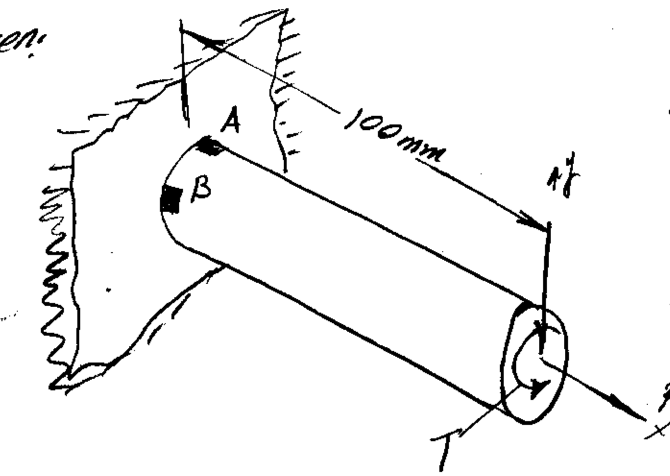
$$= \sqrt{\frac{(11.1 - 3.86)^2 + (3.86 - 0)^2 + (11.1)^2}{2}}$$

$$= 9.76\,ksi$$

$$\sigma_{eff} = \frac{S_{yp}}{N} \Rightarrow N = \frac{S_{yp}}{\sigma_{eff}}$$

$$= \frac{44\,ksi}{9.76\,ksi} = \underline{\underline{4.51}}$$

Given:



Dia. = 20 mm

P = 8 kN

F = 0.55 kN

T = 30 N.m

Mat'l ≡ AISI 1006 cold-drawn steel

Find: Safety factor at points A and B.

<u>Consider point A.</u>

$$\sigma_{xx} = \frac{Mc}{I} + \frac{P}{A} \qquad I = \frac{\pi}{64} D^4 = \frac{\pi}{64}(0.020)^4$$

$$= 8 \times 10^{-9}\, m^4$$

$$c = 0.010\, m$$

$$M = F \cdot l$$

$$= 0.55\, kN \cdot 0.10\, m \qquad A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.020)^2$$

$$= 3.14 \times 10^{-4}\, m^2$$

$$= 55\, N \cdot m$$

$$\Rightarrow \sigma_{xx} = \frac{(55\, N.m)(0.010\, m)}{8 \times 10^{-9}\, m^4} + \frac{8{,}000\, N}{3.14 \times 10^{-4}\, m^2}$$

$$= 68.8\, MPa + 25.5\, MPa = 94.3\, MPa$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{Tc}{2I} = \frac{(30 N.m)(0.010 m)}{2(8 \times 10^{-9}\, m^4)}$$

$$= 18.8\, MPa$$

$$\sigma_1 = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{94.3}{2} + \sqrt{\left(\frac{94.3}{2}\right)^2 + 18.8^2}$$

$$= 97.9\, MPa$$

$$\sigma_2 = \frac{\sigma_{xx}+\sigma_{yy}}{2} - \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{94.3}{2} - 50.8 = -3.65 \text{ MPa}$$

Ordering the principal stresses from largest to smallest yields

$\sigma_1 = 97.9$ MPa , $\sigma_2 = 0$, $\sigma_3 = -3.65$ MPa

$$\sigma_{eff} = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2}{2}}$$

$$= \sqrt{\frac{(97.9)^2 + (3.65)^2 + (97.9+3.65)^2}{2}}$$

$$= 99.8 \text{ MPa}$$

$S_{yt} = 280$ MPA

$$\sigma_{eff} = \frac{S_{yt}}{N} \Rightarrow N = \frac{S_{yt}}{\sigma_{eff}} = \frac{280 \text{ MPa}}{99.8 \text{ MPa}}$$

$$\underline{\underline{N = 2.81}}$$

Consider point B.

$$\sigma_{xx} = \frac{P}{A} = \frac{8{,}000 N}{3.14 \times 10^{-4} m^2} = 25.5 MPa$$

$$\tau_{xy} = \frac{Tc}{J} + \frac{VQ}{It}$$

$$Q = \frac{\pi}{8} D^2 \cdot \frac{2D}{3\pi}$$

$$= \frac{D^3}{12} = \frac{(0.020 m)^3}{12} = 6.67 \times 10^{-7} m^3$$

$$t = 0.020 m$$

$$\tau_{xy} = \frac{(30 N \cdot m)(0.010 m)}{2(8 \times 10^{-9} m^4)} + \frac{(550 N)(6.67 \times 10^{-7} m^3)}{(8 \times 10^{-9} m^4)(0.020 m)}$$

$$= 18.8 MPa + 2.29 MPa = 21.1 MPa$$

$$\sigma_1 = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$= 37.4 MPa$$

$$\sigma_2 = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$= -11.9 MPa$$

Arranging the principal stresses in largest to smallest order, $\Rightarrow \sigma_1 = 37.4\,MPa,\ \sigma_2 = 0.,\ \sigma_3 = -11.9\,MPa$

$$\sigma_{eff} = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}}$$

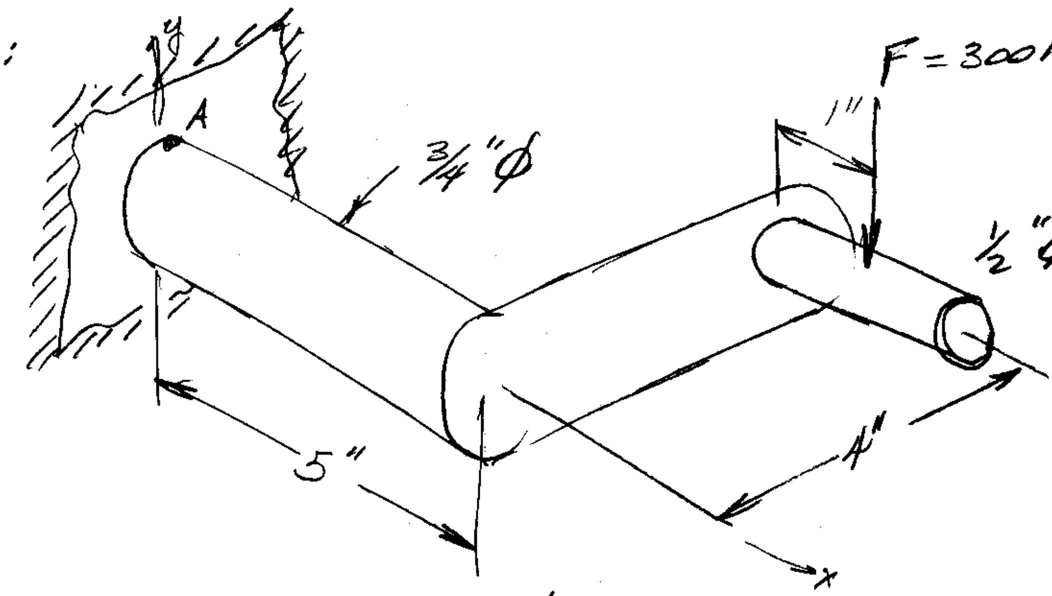
$$= \sqrt{\frac{(37.4)^2+(11.9)^2+(37.4+11.9)^2}{2}}$$

$$= 44.6\ MPa$$

$$N = \frac{S_{yt}}{\sigma_{eff}} = \frac{280\ MPa}{44.6\ MPa} = \underline{\underline{6.28}}$$

The beam is much more highly stressed at point A than at point B.

Given:



AISI 1018 Mat'l

Find: Factor of safety using MSST.

Solution:

$$\sigma_{xx} = \frac{Mc}{I} \qquad M = F(6\,in)$$

$$= (300\,lb)(6\,in) = 1800\ in\cdot lb$$

$$C = 0.375\,in$$

$$I = \frac{\pi}{64} D^4 = \frac{\pi}{64}(0.75)^4 = 1.55\times10^{-2}\,in^4$$

$$\sigma_{xx} = \frac{(1800\,in\cdot lb)(0.375\,in)}{1.55\times10^{-2}\,in^4} = \underline{\underline{43.5\ ksi}}$$

$$\tau_{xz} = -\frac{TC}{J} = -\frac{(300\,lb)(4\,in)(0.375\,in)}{2(1.55\times10^{-2}\,in^4)} = -14.5\ ksi$$

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_z}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= 47.9\ ksi$$

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_z}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = -4.39\,ksi$$

Ordering the principal stresses largest to smallest

$\Rightarrow \sigma_1 = 47.9\ ksi;\ \sigma_2 = 0,\ ,\ \sigma_3 = -4.39\ ksi$

$$N = \frac{S_{yt}}{|\sigma_1 - \sigma_3|} = \frac{S_{yt}}{|47.9 + 4.39|} = \frac{32.0\ ksi}{52.3\ ksi} = 0.611$$

$N = 0.611 \Rightarrow$ material is above its yield strength