The University of Tennessee at Martin

MARTIN School of Engineering

Steady Load Failure Theories – Comparison with Experimental Data

Lecture 7

Engineering 473
Machine Design

MARTIN

Important Historical Studies of Failure Theories

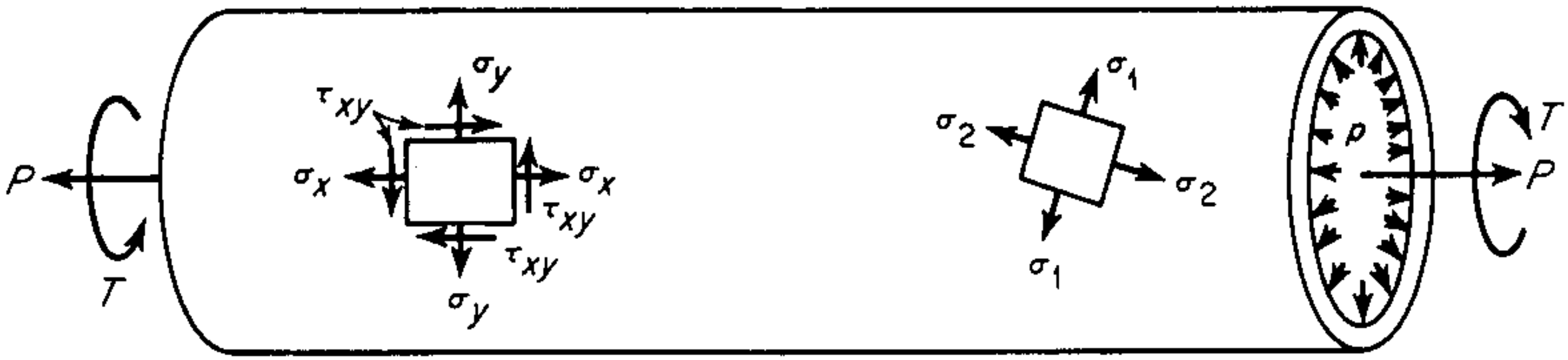
1864 Tresca developed Maximum Shear Stress Theory while measuring loads required to extrude metal through dies of various shapes.

1928 von Mises publishes the Maximum Distortion Energy Theory

1926 Lode publishes comparison of Tresca and von Mises Theories

1931 Repeat Lode experiments with better technique

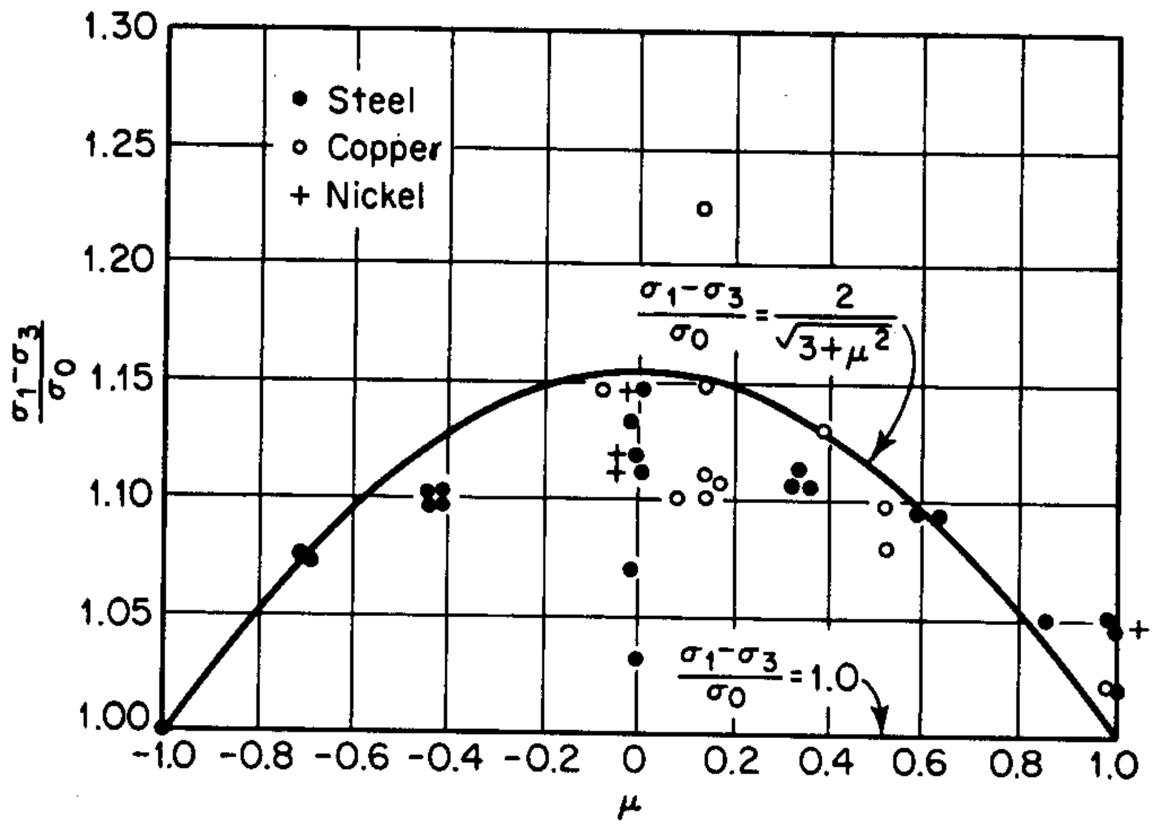
Experimental Test Specimen



Thinned walled cylinder loaded with an internal pressure, axial force, and a torsional moment.

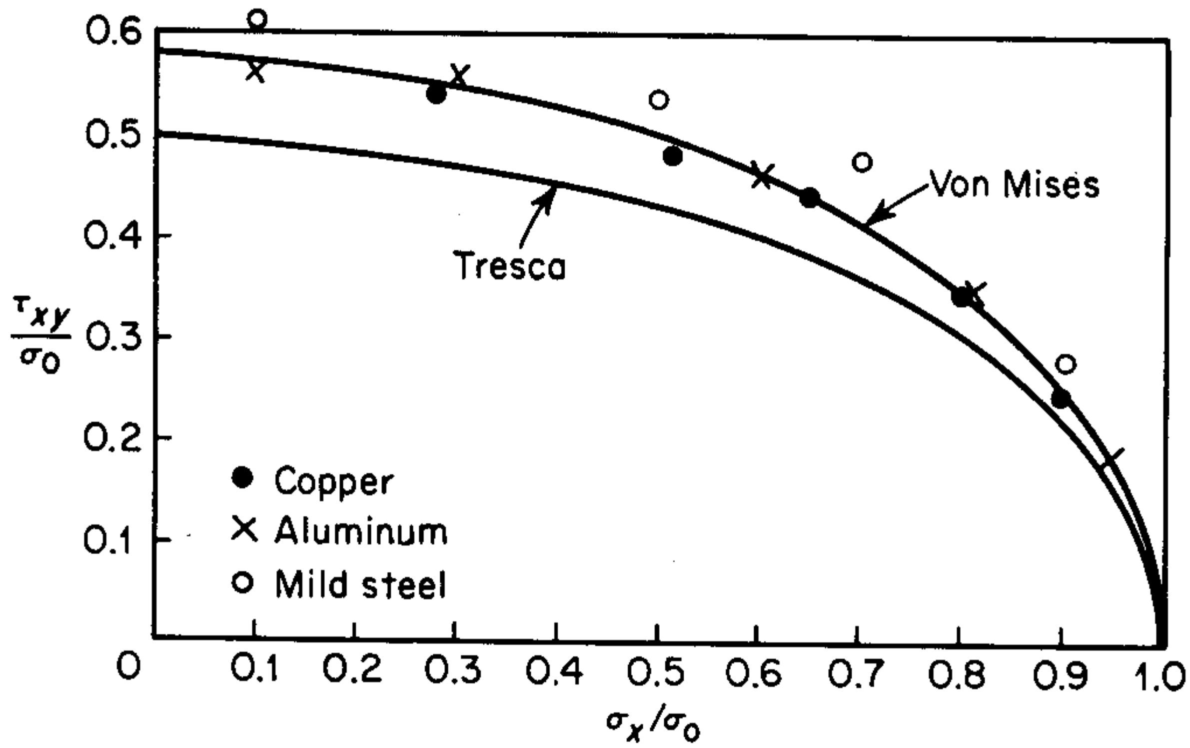
Mendelson, Fig. 6.1.1

Lode's Data



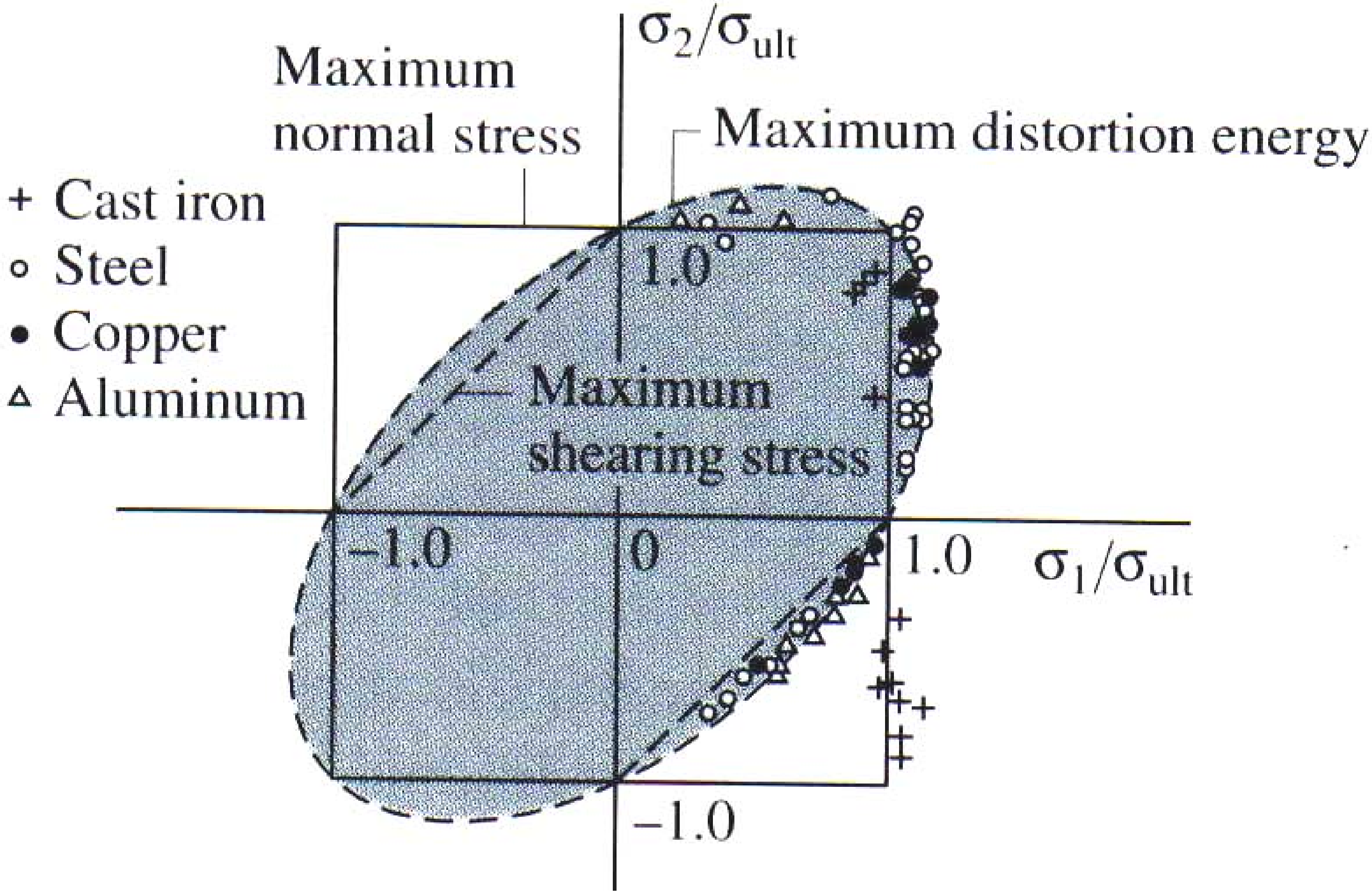
Mendelson, Fig. 6.4.1

Taylor and Quinney Data



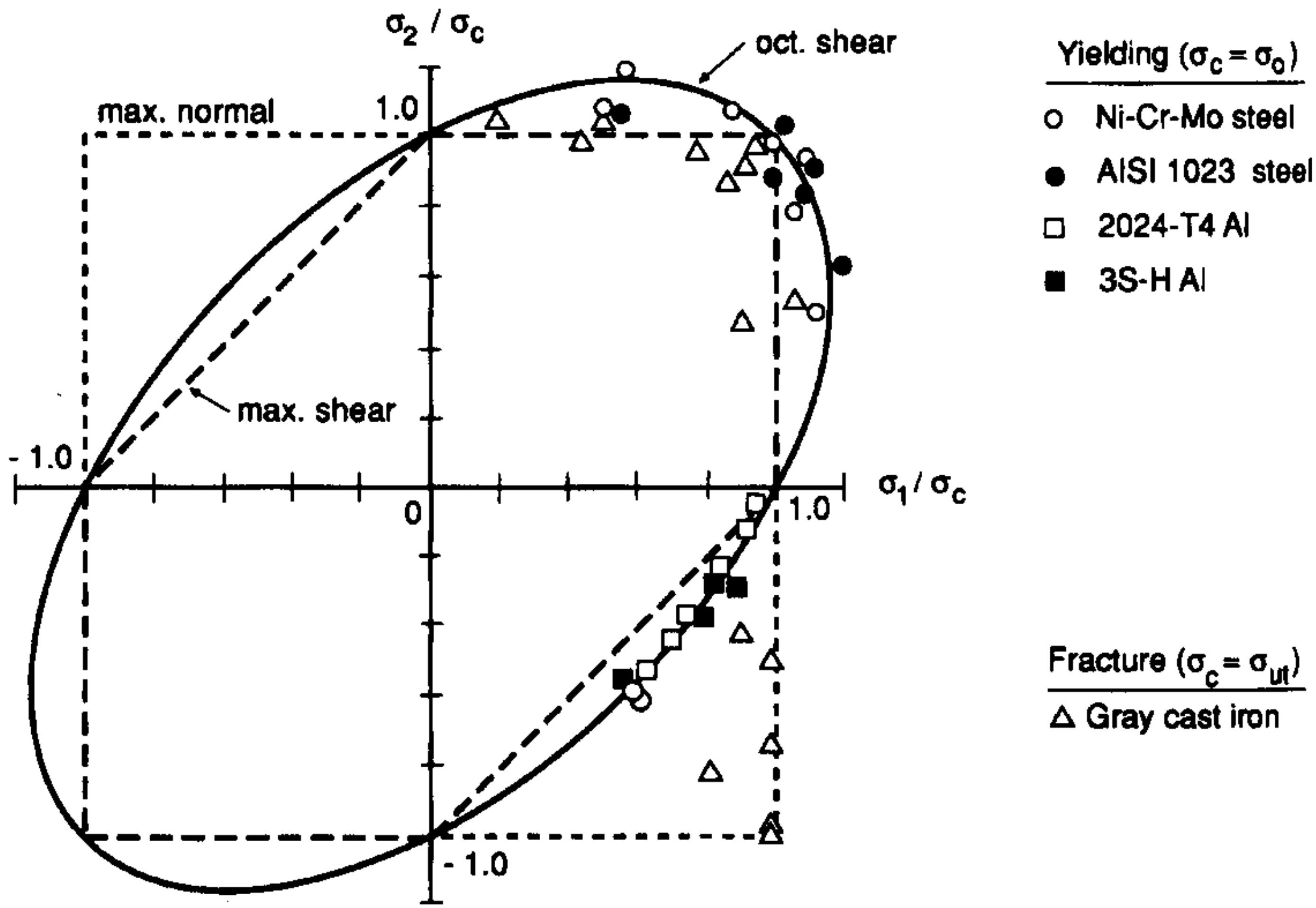
Mendelson, Fig. 6.4.3

Additional Test Results



Hamrock, Fig. 6-17

More Test Results



Dowling, Fig. 7-11

Conclusions

- Both the **Distortion Energy Theory** and the **Maximum Shear Stress Theory** provide reasonable estimates for the onset of yielding in the case of static loading of ductile, homogeneous, isotropic materials whose compressive and tensile strengths are approximately the same.

- Both the **Distortion Energy Theory** and the **Maximum Shear Stress Theory** predict that the onset of yield is independent of the hydrostatic stress. This agrees reasonably well with experimental data for moderate hydrostatic pressures.

Conclusions
(Continued)

- Both the **Distortion Energy Theory** and the **Maximum Shear Stress Theory** under predict the strength of brittle materials loaded in compression. Brittle materials often have much higher compressive strengths than tensile strengths.

- The **Distortion Energy Theory** is slightly more accurate than the **Maximum Shear Stress Theory**. The Distortion Energy Theory is the yield criteria most often used in the study of classical plasticity. Its continuous nature makes it more mathematically amenable.

Industry Standards and Codes

- **The American Society of Mechanical Engineers base the <u>ASME Boiler and Pressure Vessel Code</u> on the Maximum Shear Stress Theory.**
- **The American Institute of Steel Construction does not use either in the <u>Manual of Steel Construction</u>. Buildings, bridges, etc. are dominated by normal stresses and buckling type failures.**
- **The American Society of Civil Engineers use the Distortion Energy Theory in <u>Design of Steel Transmission Pole Structures</u>.**
- **There is no single standard that applies to the design of machine components. Standard industry practice is to use either the Distortion Energy Theory or Maximum Shear Stress Theory with an appropriate safety factor.**

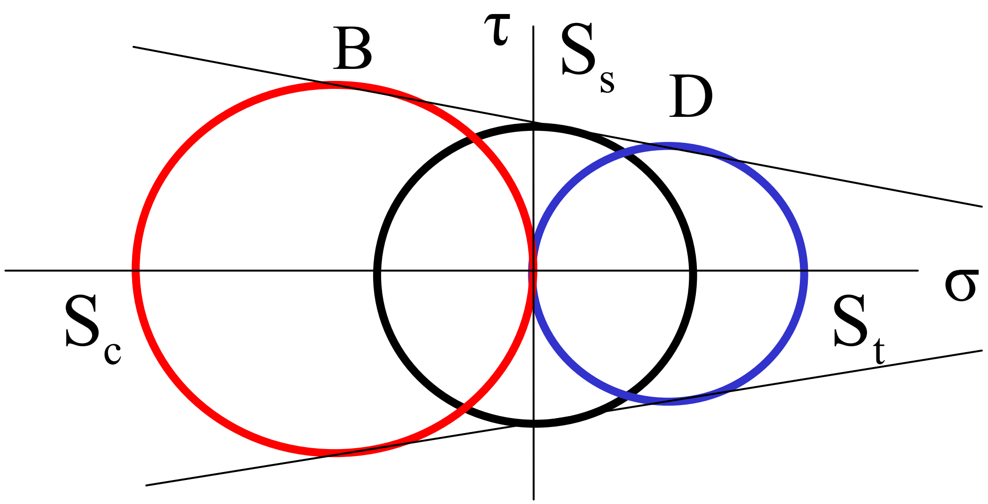
Failure Versus Yielding

- The high stresses around stress concentration factors are often very localized, and the local yielding will cause a redistribution of stresses to adjacent material. In many cases the local yielding will not cause a machine component to fail under steady load conditions.

- It is common to differentiate between local yielding and gross yielding through the thickness of a member.

- Local yielding may lead to early fatigue failure, and stress concentration effects must always be considered in fatigue calculations.

Internal Friction Theory



Postulate: For any stress state that creates a Mohr's circle that is tangent to the line between points B&D, the stresses and strengths are related by the equation

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1, \quad \text{where } \sigma_1 > \sigma_2 > \sigma_3.$$

Comparison with Maximum Shear Stress Theory

Internal Friction Theory

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1, \quad \text{where } \sigma_1 > \sigma_2 > \sigma_3.$$

Maximum Shear Stress Theory

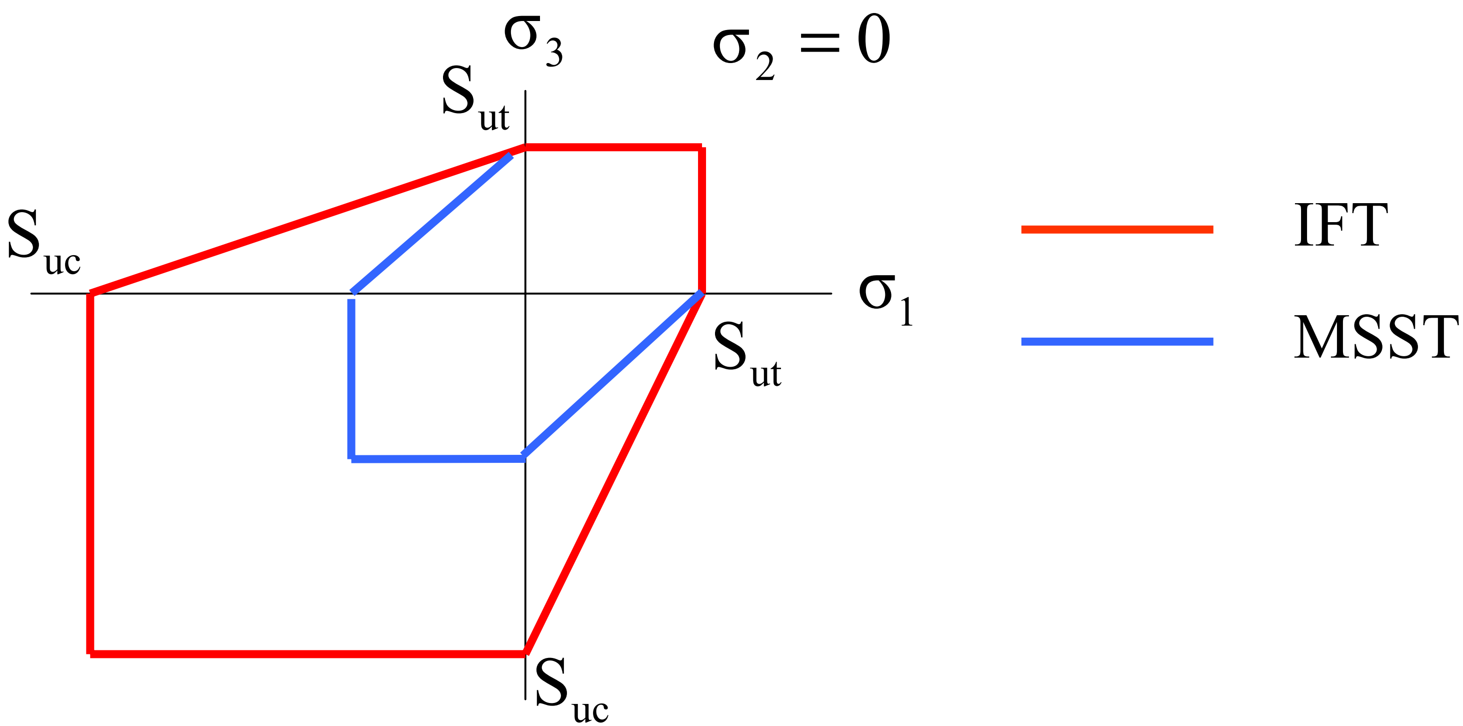
$$S_t = S_c$$

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1, \quad \text{where } \sigma_1 > \sigma_2 > \sigma_3,$$

$$\frac{\sigma_1 - \sigma_3}{S_y} = 1$$

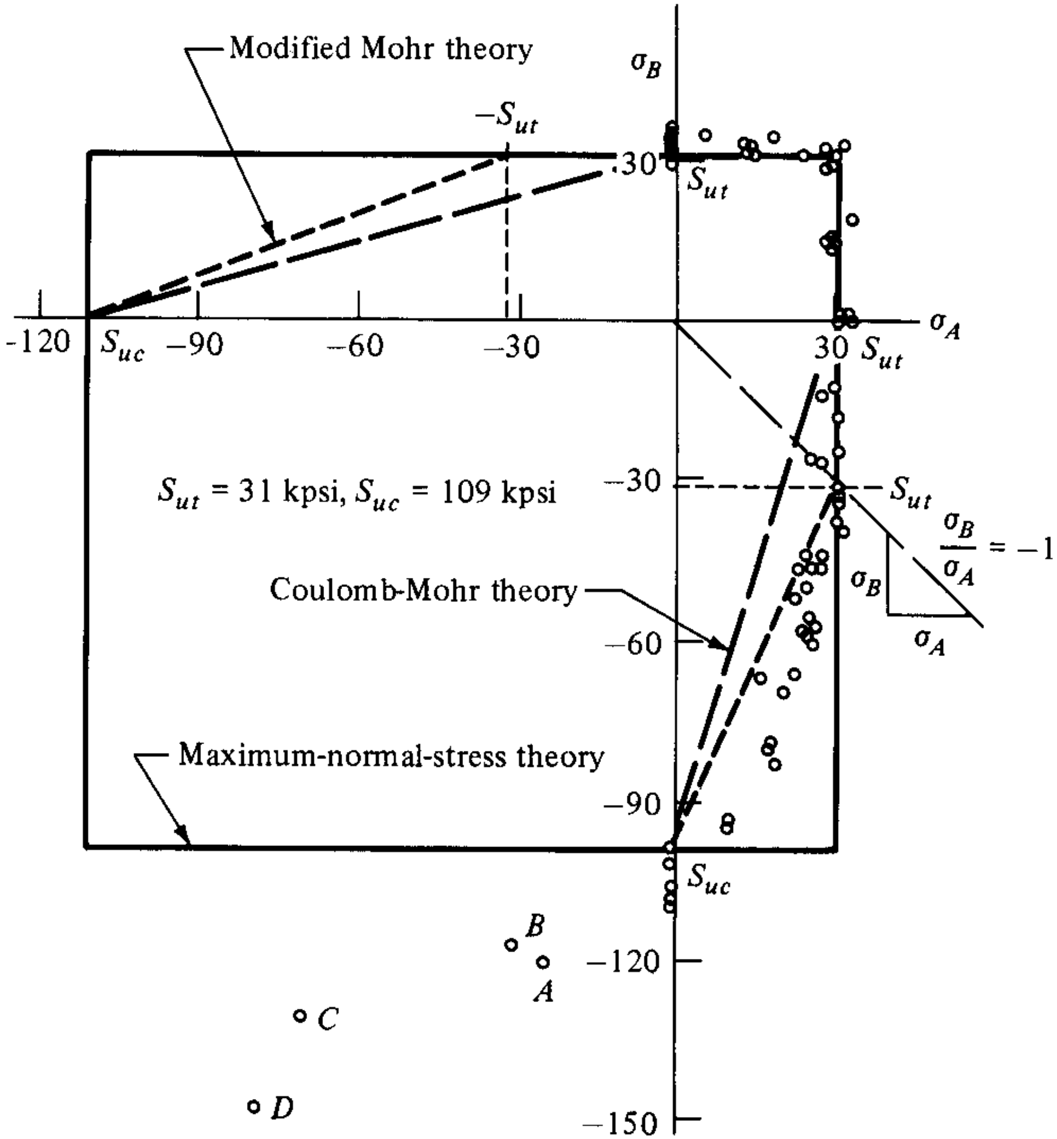
Note that the IFT is a generalization of the MSST. The MSST is limited to materials in which the tensile and compressive yield strengths are approximately equal.

Plane Stress Condition



Whenever the stress state is within the polygon, the material will not fail.

Comparison with Test Data



Colomb-Mohr Theory is the IFT

Shigley, Fig. 6-28

Brittle Material Failure Summary

- Brittle materials typically have significantly different compressive and tensile strengths.
- The **Internal Friction Theory** or **Modified Internal Friction Theory** may be used to estimate the failure state.
- For some materials the **Modified Internal Friction Theory** may provide a slightly more accurate estimate.

Safety Factors

N = **Safety Factor**

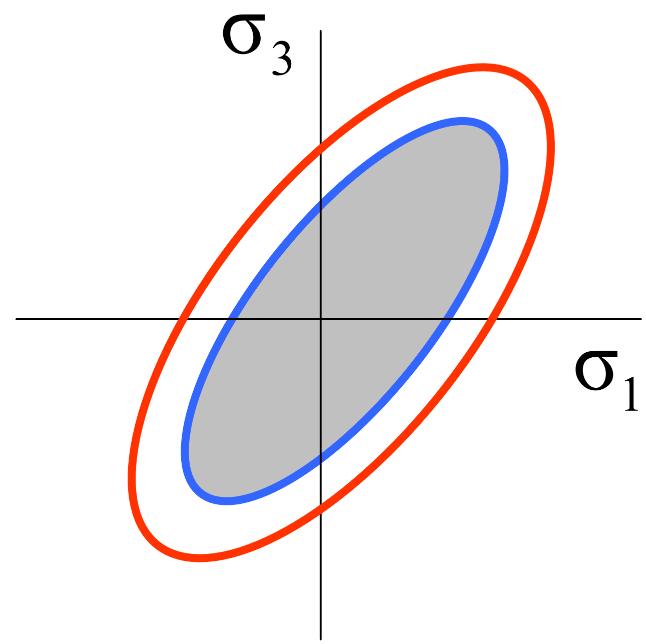
$$\frac{\sigma_{eff}}{S_y} = \frac{1}{N}$$ DET

$$\frac{\sigma_1}{S_t} = \frac{\sigma_3}{S_t} = \frac{1}{N}$$ 1st Quadrant IFT

$$\frac{\sigma_1}{-S_c} = \frac{\sigma_3}{-S_c} = \frac{1}{N}$$ 3rd Quadrant IFT

$$-\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{N}$$ 2nd Quadrant IFT

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{N}$$ 4th Quadrant IFT



Reduced area of allowable stress states.

Design Margins

$$\frac{\sigma_{eff}}{S_y} = \frac{1}{N}$$

$$S_y - \sigma_{eff} N = 0$$

$$\text{Margin} \equiv M = \frac{S_y - \sigma_{eff} N}{S_y}$$

$$M = 1 - \frac{\sigma_{eff} N}{S_y}$$

- For a stress state to be acceptable, the **margin** must be positive.
- A negative margin indicates that the design objective hasn't been met.
- Provides a measure of how close a stress state is to the design maximum.
- Design Margins are reported for all NASA projects.

Assignment

A hot-rolled bar has a minimum yield strength in tension and compression of 44 kpsi. Find the factors of safety for the MSST and DET failure theories for the following stress states.

$$(a)\ \sigma_{xx} = 9\text{ kpsi},\ \sigma_{yy} = -5\text{ kpsi}$$

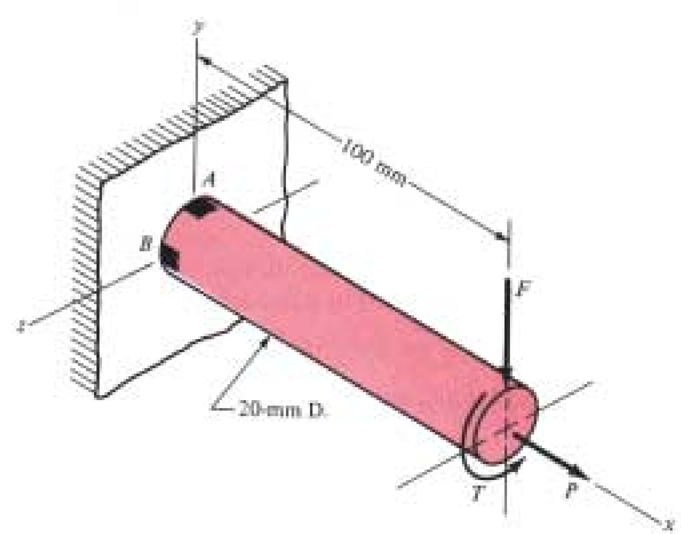
$$(b)\ \sigma_{xx} = 12\text{ kpsi},\ \tau_{xy} = 3\text{ kpsi ccw}$$

$$(c)\ \sigma_{xx} = -4\text{ kpsi},\ \sigma_{yy} = \text{-}9\text{ kpsi},\ \tau_{xy} = 5\text{ kpsi cw}$$

$$(d)\ \sigma_{xx} = 11\text{ kpsi},\ \sigma_{yy} = 4\text{ kpsi},\ \tau_{xy} = 1\text{ kpsi cw}$$

Assignment
(Continued)

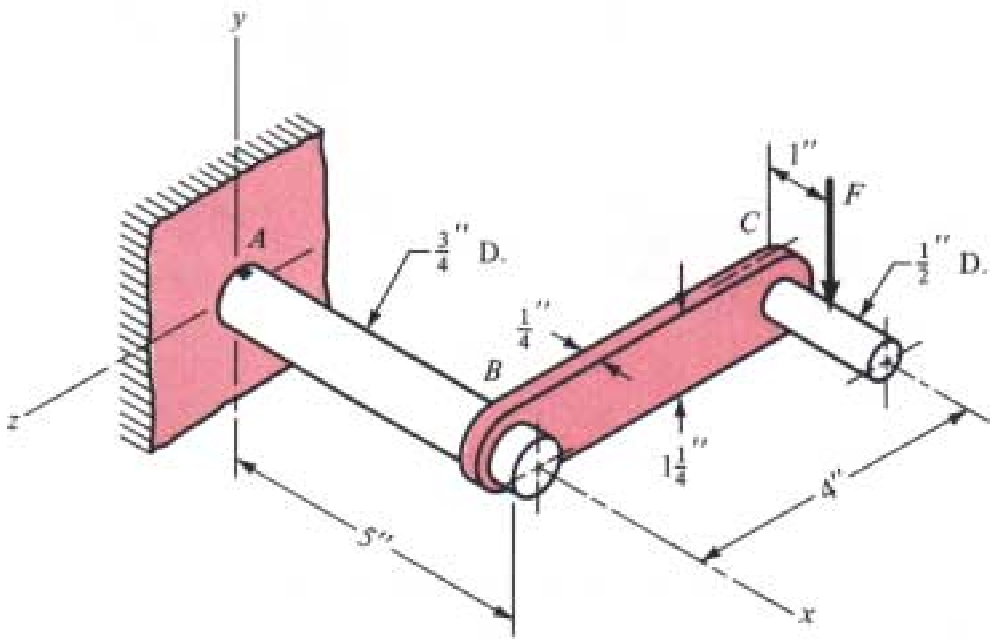
This problem illustrates that the factor of safety for a machine element depends on the particular point selected for analysis. You are to compute factors of safety, based upon the distortion-energy theory, for stress elements at A and B of the member shown in the figure. The bar is made of AISI 1020 cold-drawn steel and is loaded by the forces F=0.55 kN, P=8.0 kN, and T=30 Nm.



Shigley, Problem 6-6

Assignment
(Continued)

The figure shows a crank loaded by a force F=300 lb which causes twisting and bending of the 0.75 in diameter shaft fixed to a support at the origin of the reference system. The material is hot-rolled AISI 1020 steel. Using the maximum-shear-stress theory, find the factor of safety based on the stress state at point A.



Shigley, Problem 6-8