



*The University of Tennessee at Martin*

**MARTIN**

**School of Engineering**

# **Introduction to Fracture Mechanics**

## **Lecture 8**

**Engineering 473**

**Machine Design**



# Fracture Mechanics

“...every structure contains small flaws whose size and distribution are dependent upon the material and its processing. These may vary from nonmetallic inclusions and micro voids to weld defects, grinding cracks, quench cracks, surface laps, etc.”

The objective of a **Fracture Mechanics** analysis is to determine if these small flaws will grow into large enough cracks to cause the component to fail catastrophically.

**T.J. Dolan, Preclude Failure: A Philosophy for Material Selection and Simulated Service Testing, SESA J. Exp. Mech., Jan. 1970.**

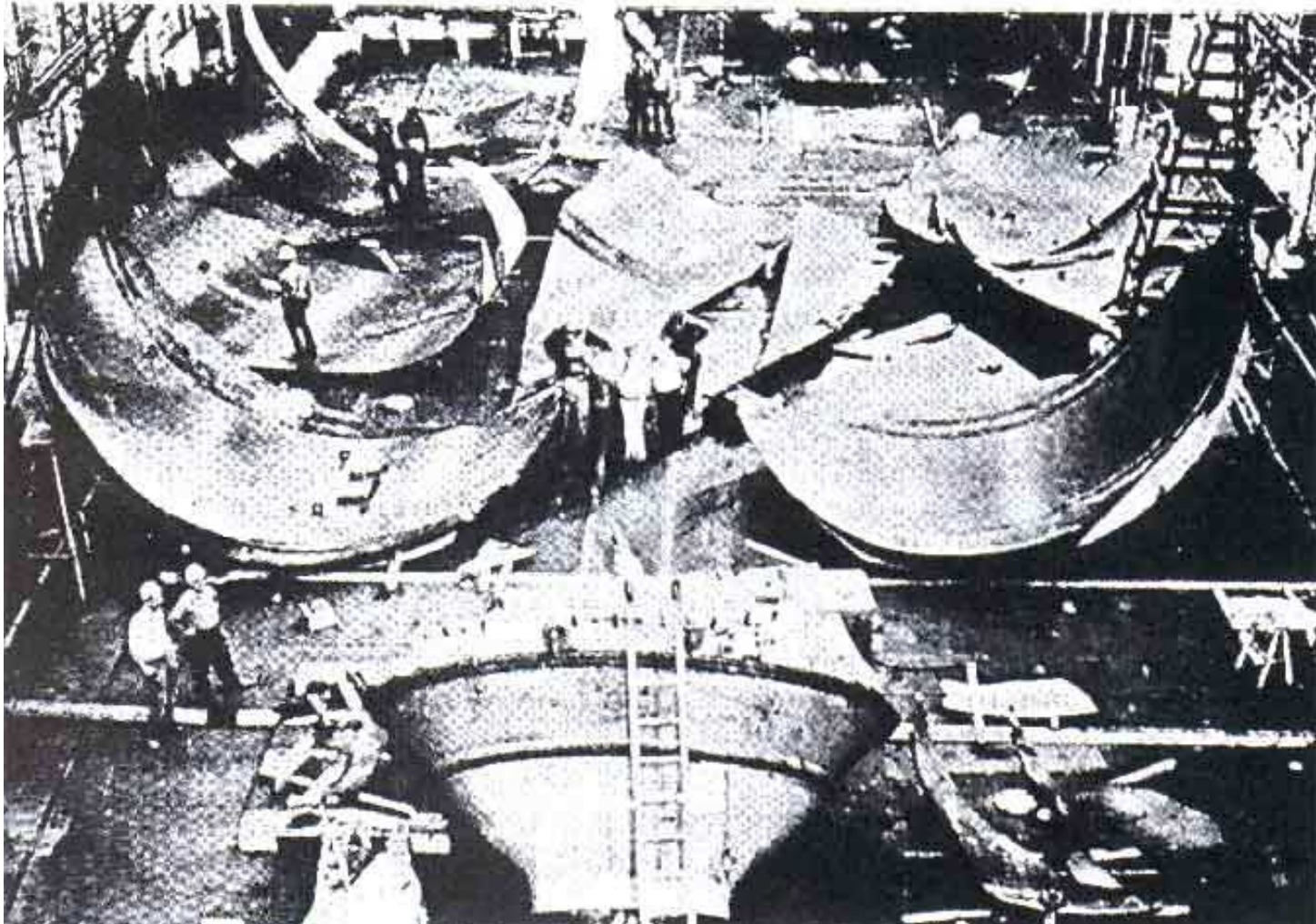
# WW II Tanker Failure



Small cracks and defects can lead to catastrophic failure of large structural systems.

Norton, Fig. 5-13

# Rocket Case Failure



Norton, Fig. 5-14

# Stress State at Plane Crack Tip

$$\sigma_x = \frac{K}{\sqrt{2\pi \cdot r}} \cos\left(\frac{\theta}{2}\right) \left[ 1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right] + \dots$$

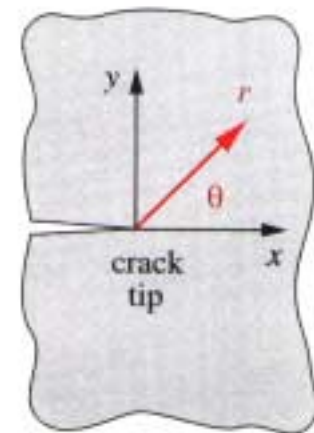
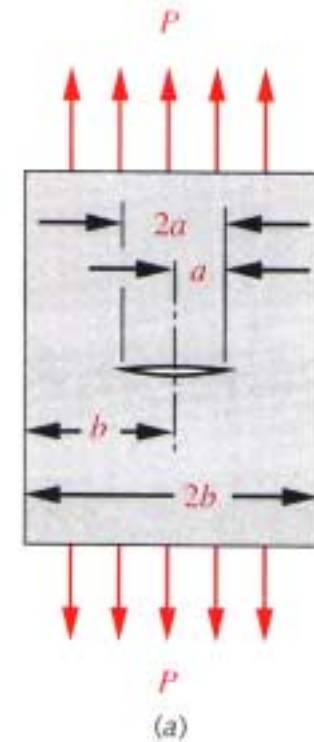
$$\sigma_y = \frac{K}{\sqrt{2\pi \cdot r}} \cos\left(\frac{\theta}{2}\right) \left[ 1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right] + \dots$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi \cdot r}} \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) + \dots$$

$$\sigma_z = 0 \quad (\text{Plane Stress})$$

$$\sigma_z = \nu(\sigma_x + \sigma_y) \quad (\text{Plane Strain})$$

$$\tau_{yz} = \tau_{zx} = 0$$



# Stress Intensity Factor

$K \equiv$  Stress Intensity Factor

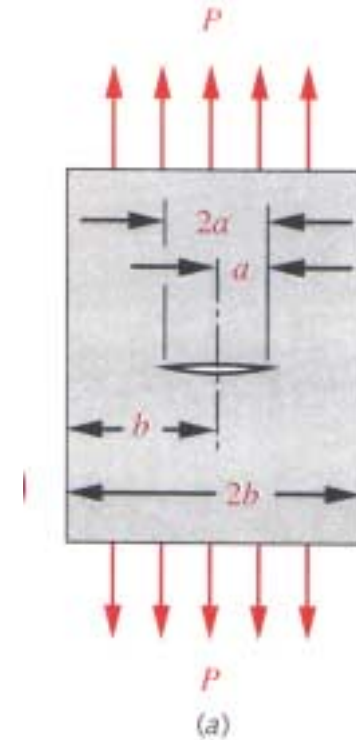
$$K = \sigma_{\text{nom}} \sqrt{\pi \cdot a}$$

for

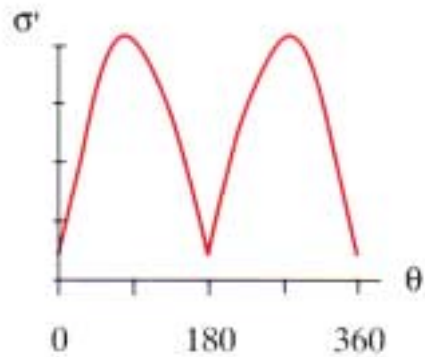
$$a \ll b$$

$\sigma_{\text{nom}} \equiv$  Stress in the absence of the crack

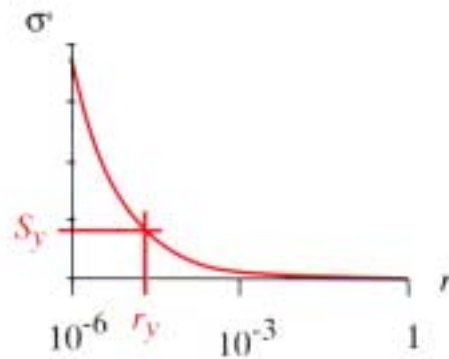
$$[k] = \frac{\text{MPa}}{\sqrt{\text{m}}} \text{ or } \frac{\text{ksi}}{\sqrt{\text{in}}}$$



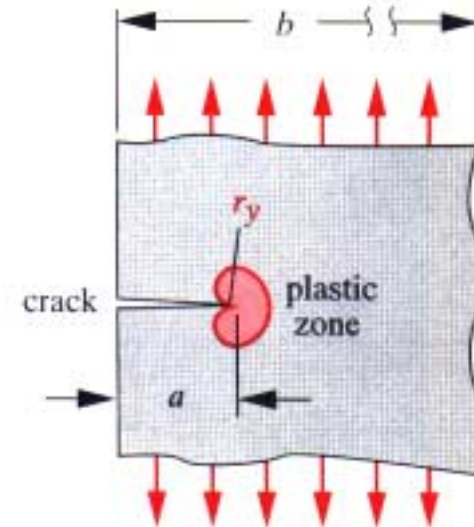
# Crack Tip Plastic Zone



(a) Von Mises stress as a function of angle around crack tip ( $r = 10E-6$  in)

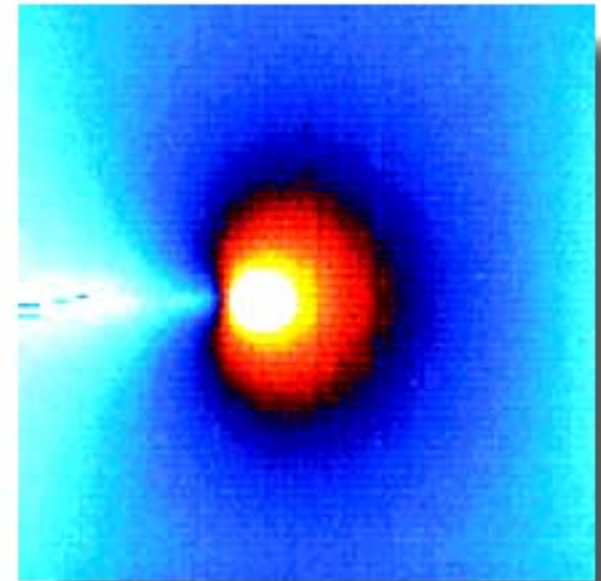
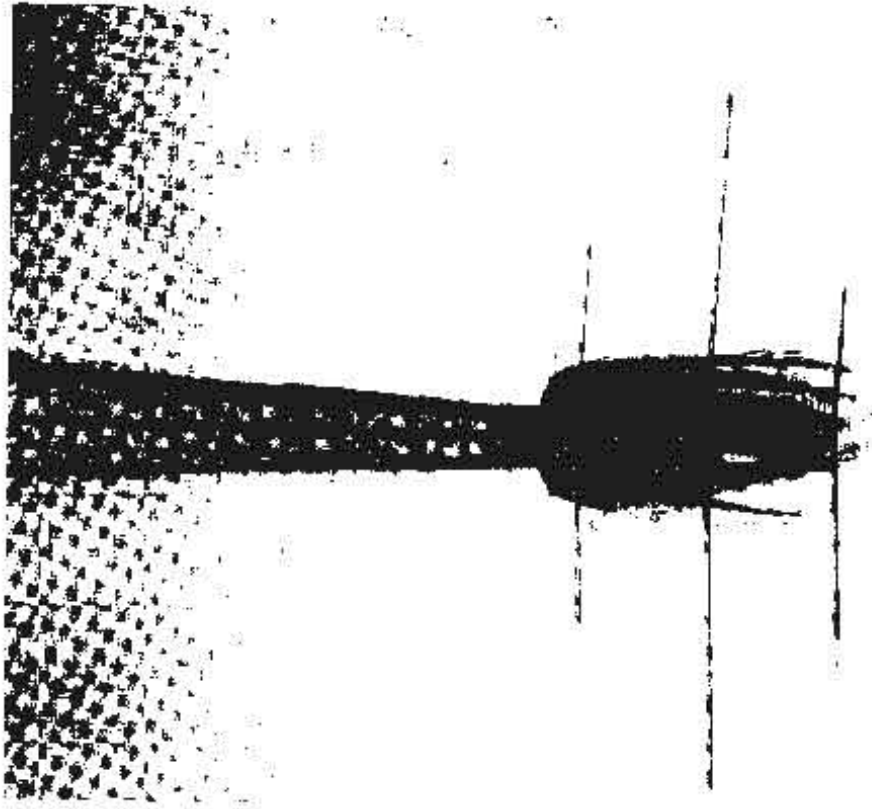


(b) Von Mises stress as a function of distance from crack tip ( $\theta = 81^\circ$ )



(c) Zone of plastic yielding around crack tip

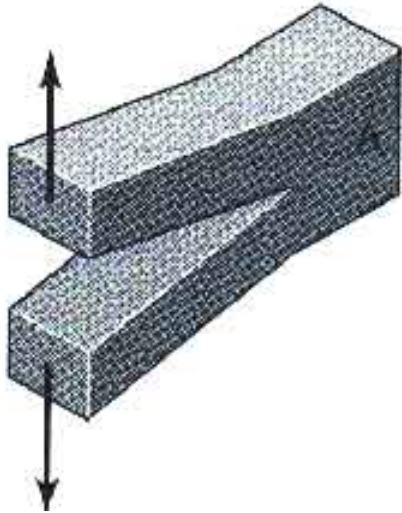
# Experimental Examples



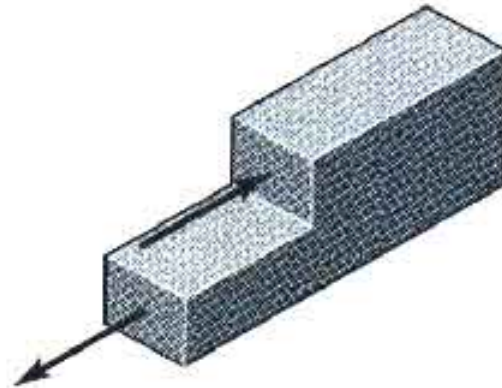
Felbeck, D.K., A.G. Atkins, *Strength and Fracture of Engineering Solids*, Prentice-Hall, 1984, Fig. 14-17.

[www.stressphotonics.com](http://www.stressphotonics.com)

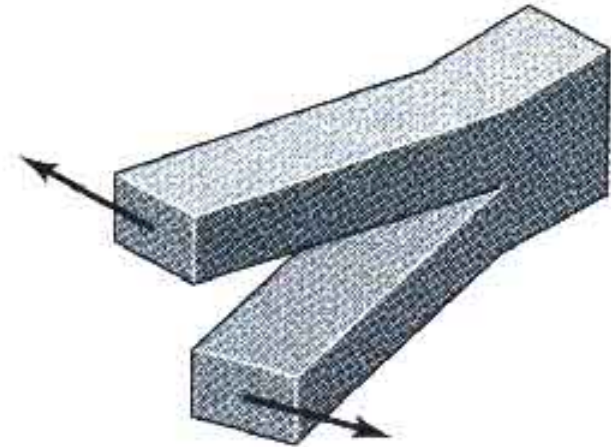
# Crack Displacement Modes



**Mode I  
Opening**



**Mode II  
Sliding**



**Mode III  
Tearing**

# Fracture Toughness

$K \equiv$  Stress Intensity Factor

$$K = \sigma_{\text{nom}} \sqrt{\pi \cdot a}$$

for

$$a \ll b$$

$\sigma_{\text{nom}} \equiv$  Stress in the  
absence of the crack

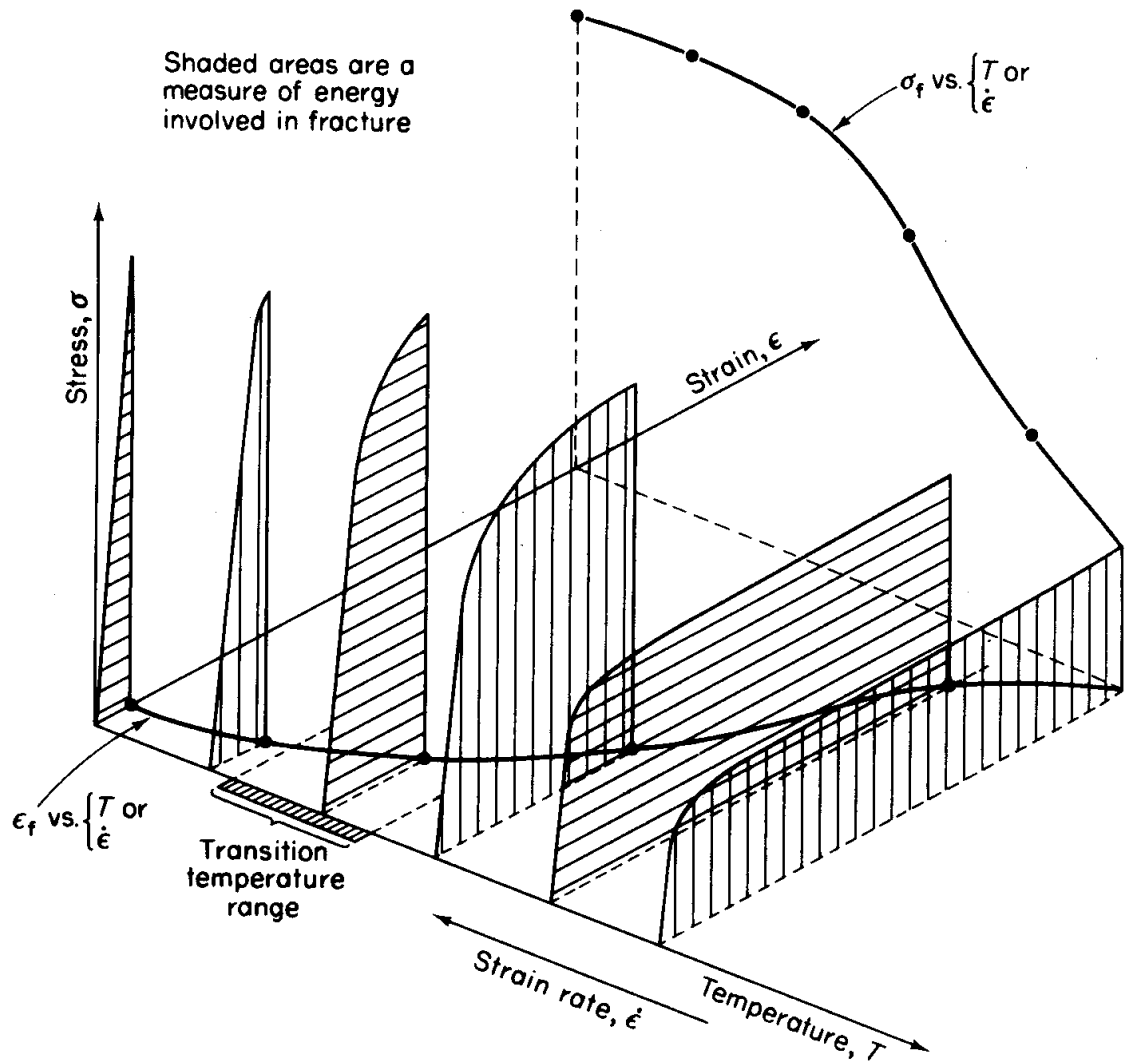
$$[k] = \frac{\text{MPa}}{\sqrt{\text{m}}} \quad \text{or} \quad \frac{\text{ksi}}{\sqrt{\text{in}}}$$

As long as the stress intensity factor  $K$  stays below a critical value called the **fracture toughness,  $K_c$** , the crack is considered stable.

If  $K$  reaches  $K_c$ , the crack will propagate and lead to sudden failure. Propagation rates can reach 1 mile/sec.

Fracture toughness is a material property.

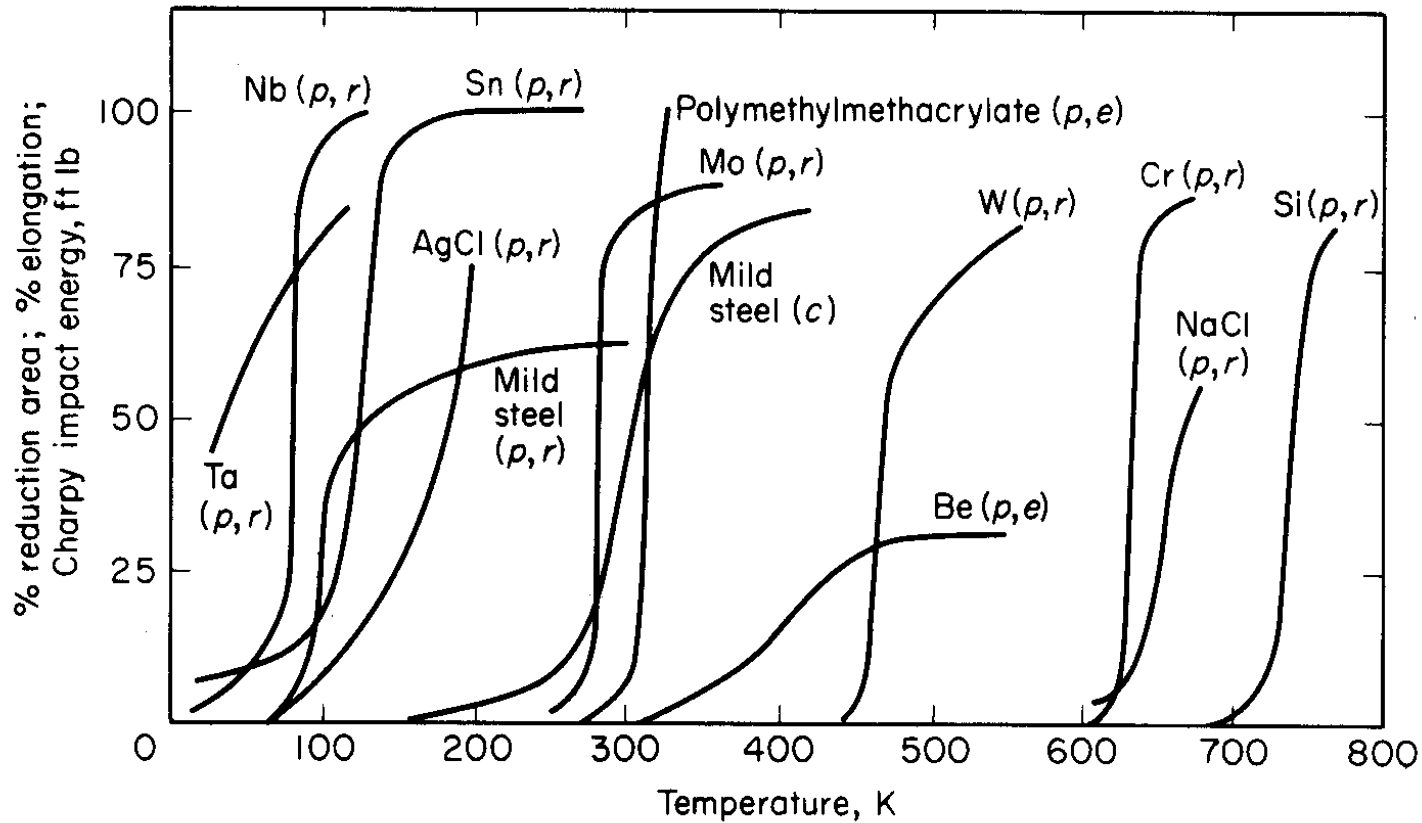
# Brittle to Ductile Transition Temperature



Low temperatures and high strain rates generally promote brittle behavior (i.e. low fracture toughness).

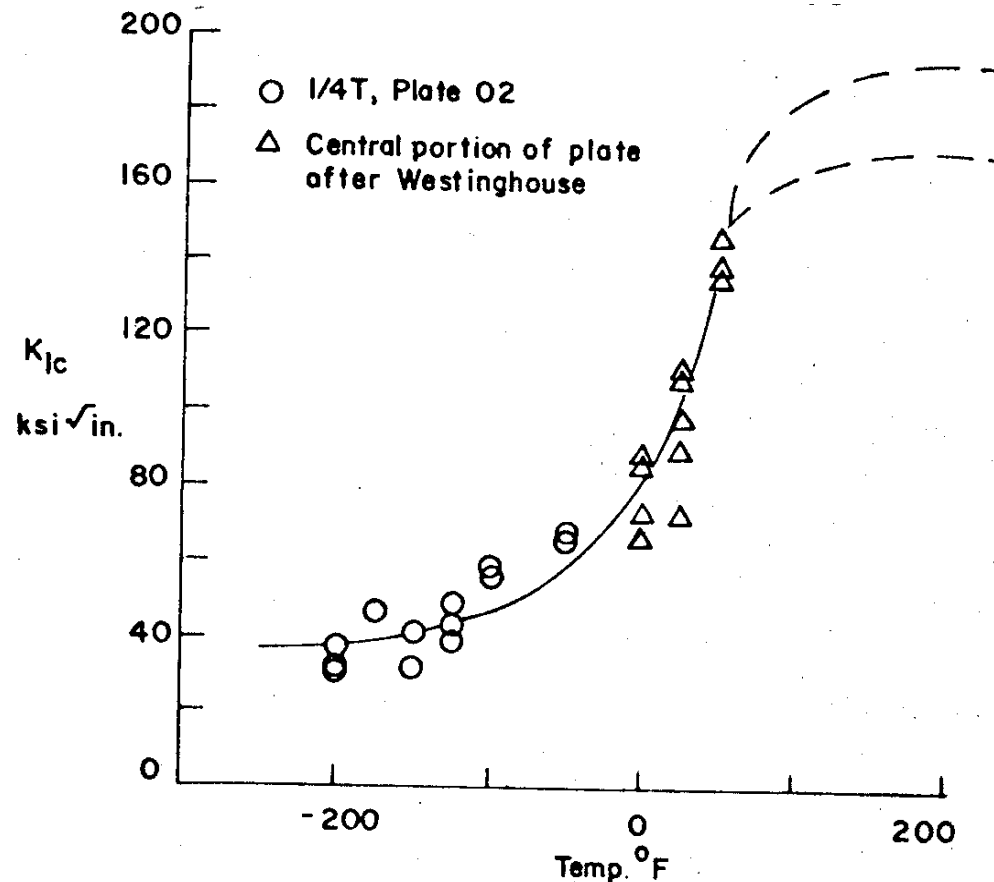
Felbeck, Fig. 14-4

# Transition Temperature Examples



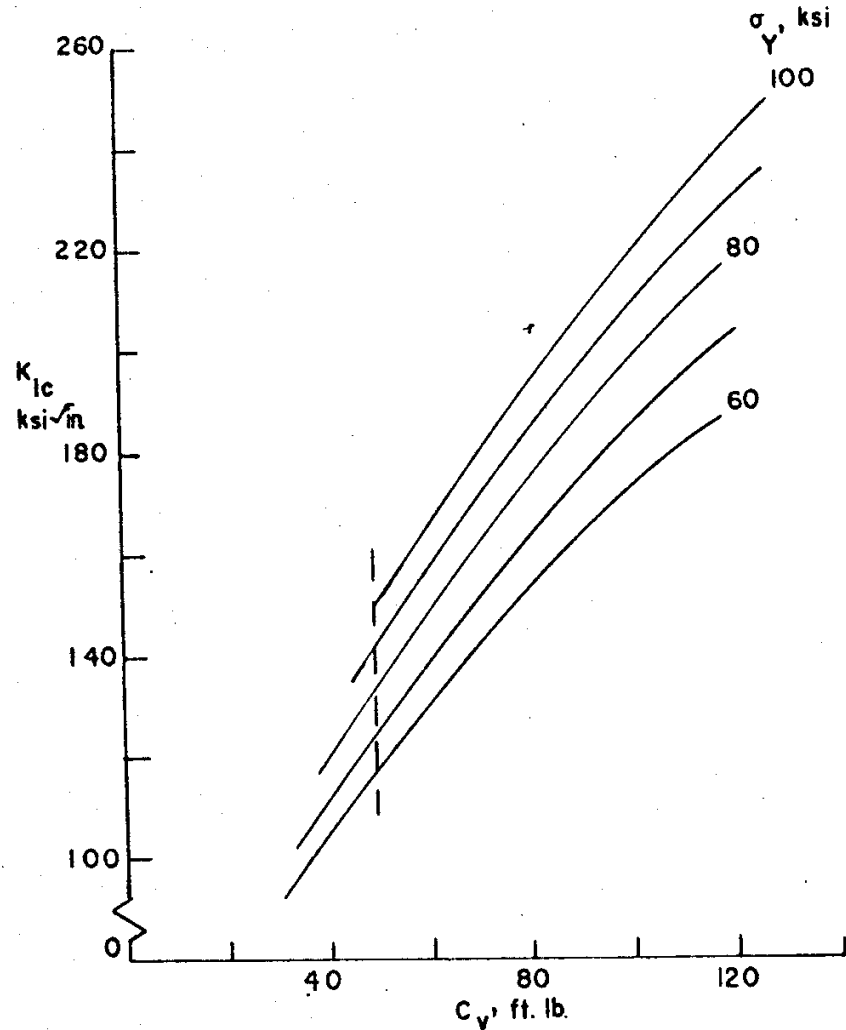
Felbeck, Fig. 14-5

# Temperature Sensitivity of $K_{IC}$



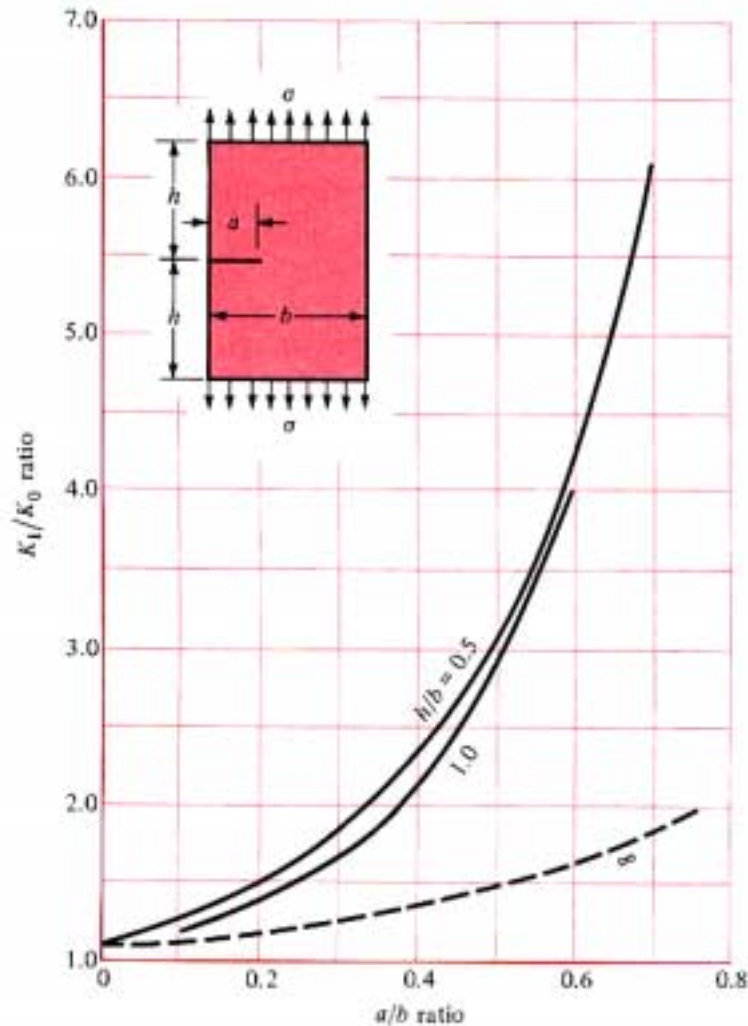
Sailors, R.H., H.T. Corten, "Relationship Between Material Fracture Toughness Using Fracture Mechanics & Transition Temperature Tests, Stress Analysis and Growth of Cracks," ASTM STP514, Am. Society of Testing Materials, 1972.

# Comparison with Charpy V-Notch Test Data



Sailors, R.H., H.T. Corten,  
“Relationship Between Material  
Fracture Toughness Using Fracture  
Mechanics & Transition Temperature  
Tests, Stress Analysis and Growth of  
Cracks,” ASTM STP514, Am. Society of  
Testing Materials, 1972.

# Stress Intensity Factors for Different Crack Geometries



Relationships between  $K_{IC}$  and other crack geometries and loading conditions may be found in text books and industry publications.

$$K_o = \sigma_{nom} \sqrt{\pi \cdot a}$$

Shigley contains several examples.

Shigley, Fig. 5-22

# Yield Failure Before Fracture

$$K_{IC} = \sigma_{nom} \sqrt{\pi \cdot a}$$

$$K_{IC} = S_y \sqrt{\pi \cdot a}$$

$$a = \frac{1}{\pi} \left( \frac{K_{IC}}{S_y} \right)^2$$

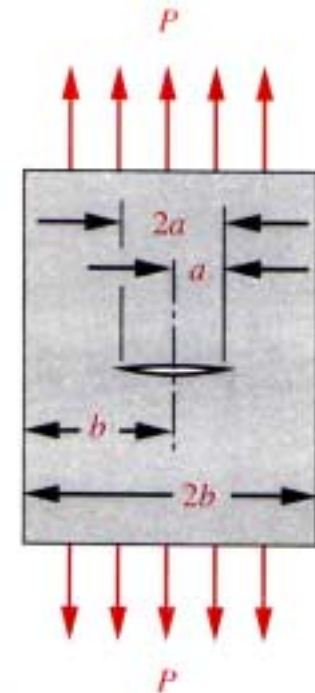
2024 Aluminum

$$K_{IC} = 26 \text{ MPa}\sqrt{\text{m}}$$

$$S_y = 455 \text{ MPa}$$

$$a = 0.001 \text{ m}$$

$$= 1 \text{ mm} = 0.04 \text{ inch}$$



**The cross section will yield before unstable fracture for any crack less than 2 mm in total length.**

# Assignment

It is determined that a high strength alloy plate has a  $\frac{1}{2}$  inch long through crack running normal to the direction of loading. Material tests indicate that the Mode I fracture toughness,  $K_{IC}$ , is  $80 \text{ ksi/in}^{1/2}$ . A stress analysis indicates that the plate will experience a steady stress of 100 ksi. Will the plate experience unstable crack propagation.

