



*The University of Tennessee at Martin*

**School of Engineering**

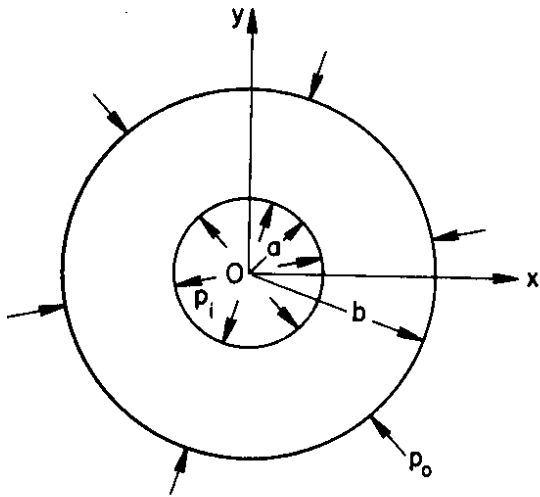
# **Compound Cylinders & Discontinuity Stresses**

## **Lecture 14**

**Engineering 473  
Machine Design**



# Lame' Equations for Thick Walled Cylinders



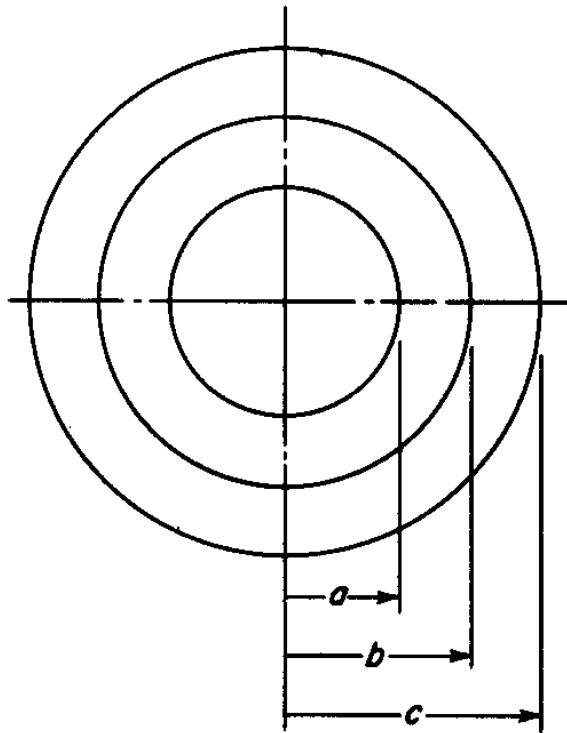
$$\sigma_r = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} - \frac{(p_i - p_o) a^2 b^2}{(b^2 - a^2) r^2}$$

$$\sigma_\theta = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{(p_i - p_o) a^2 b^2}{(b^2 - a^2) r^2}$$

$$u = \frac{1 - \nu}{E} \frac{(a^2 p_i - b^2 p_o) r}{b^2 - a^2} + \frac{1 + \nu}{E} \frac{(p_i - p_o) a^2 b^2}{(b^2 - a^2) r}$$

# Compound Cylinders

## Civil War Parrott Guns



Compound cylinders are used to increase the pressure that can be contained in cylinders.

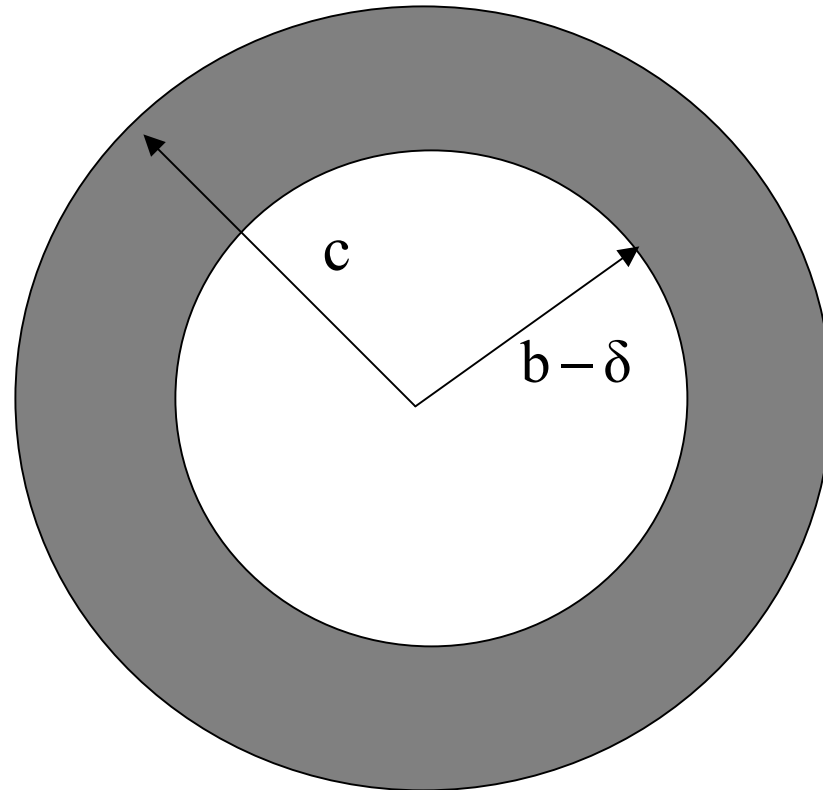
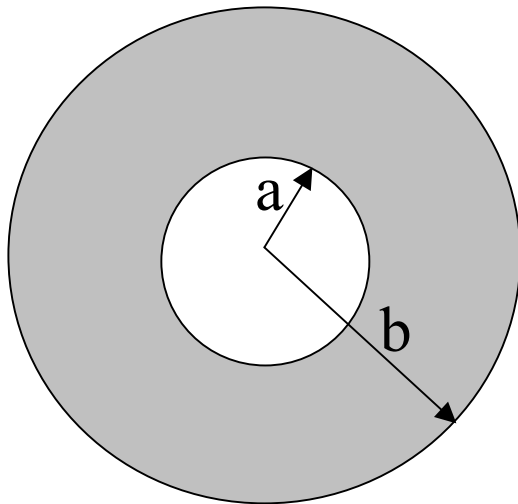


# Compound Cylinders

(Assembly)

Outer Cylinder (2)

Inner Cylinder (1)

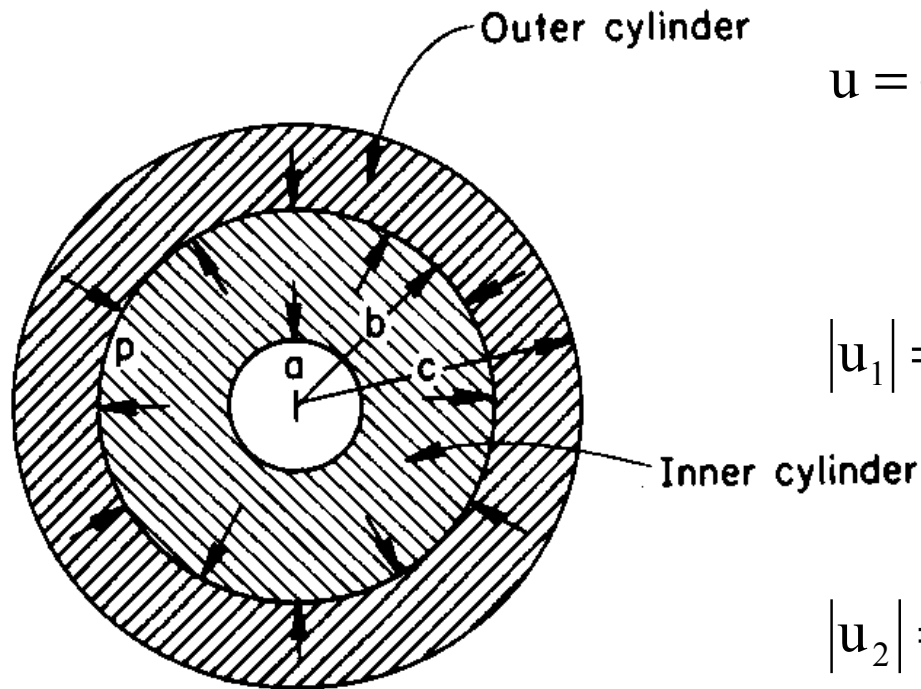


The inside diameter of cylinder 2 is undersized by a small amount (interference) and must be heated to fit over cylinder 1. This is often referred to as a shrink fit.

# Compound Cylinders

## (Interference Equations)

### Lame's Equation



$$u = \frac{1 - \nu}{E} \frac{(a^2 p_i - b^2 p_o) r}{b^2 - a^2} + \frac{1 + \nu}{E} \frac{(p_i - p_o) a^2 b^2}{(b^2 - a^2) r}$$

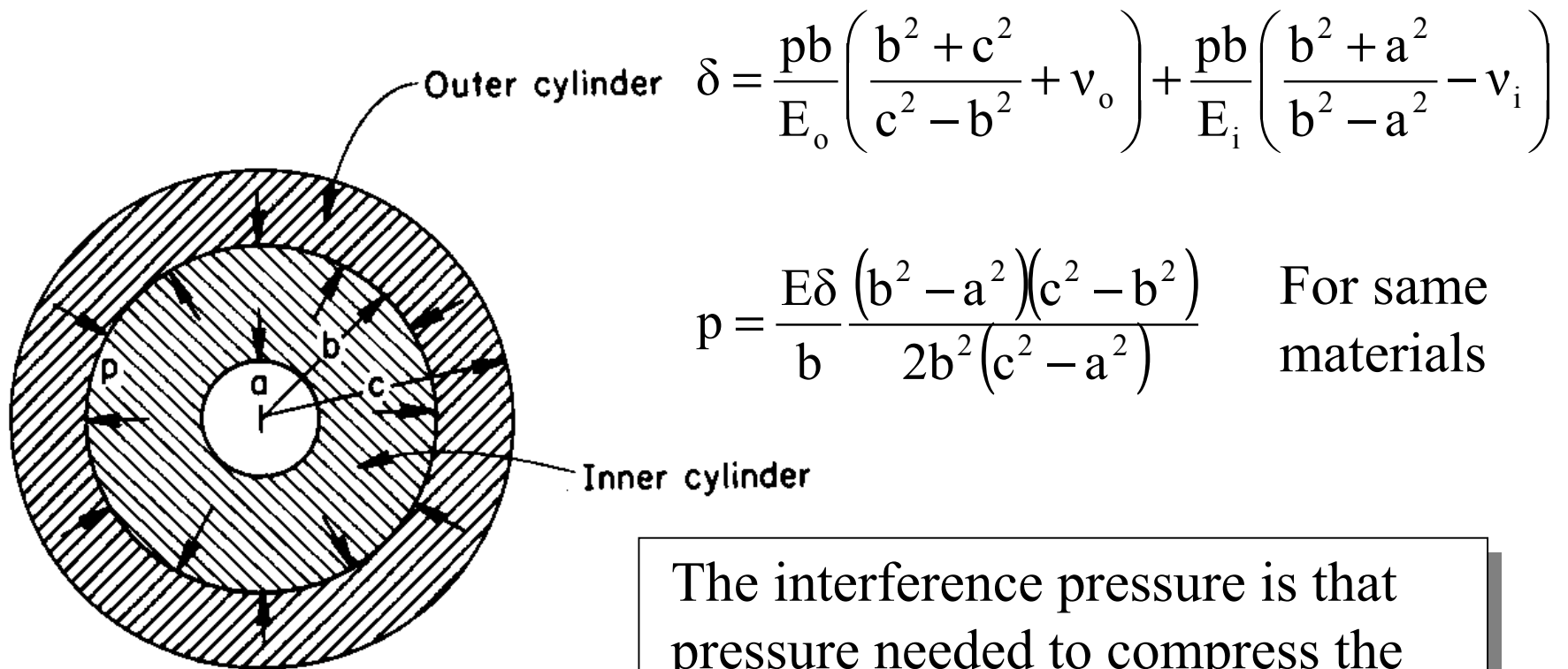
$$|u_1| = \frac{pb}{E_i} \left( \frac{b^2 + a^2}{b^2 - a^2} - \nu_i \right) \quad \text{Inner Cylinder}$$

$$|u_2| = \frac{pb}{E_o} \left( \frac{b^2 + c^2}{c^2 - b^2} + \nu_o \right) \quad \text{Outer Cylinder}$$

The interface pressure is directly proportional to the interference.

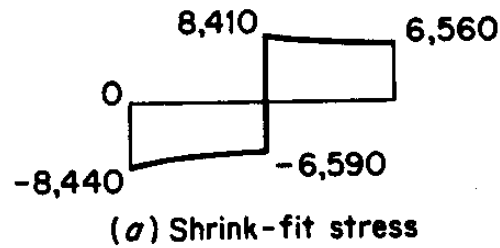
$$\delta = \frac{pb}{E_o} \left( \frac{b^2 + c^2}{c^2 - b^2} + \nu_o \right) + \frac{pb}{E_i} \left( \frac{b^2 + a^2}{b^2 - a^2} - \nu_i \right)$$

# Compound Cylinders (Interference Pressure)

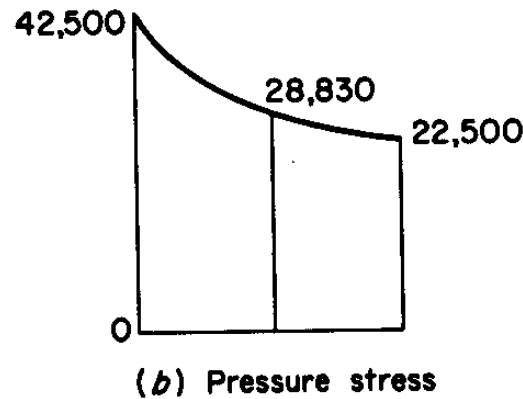
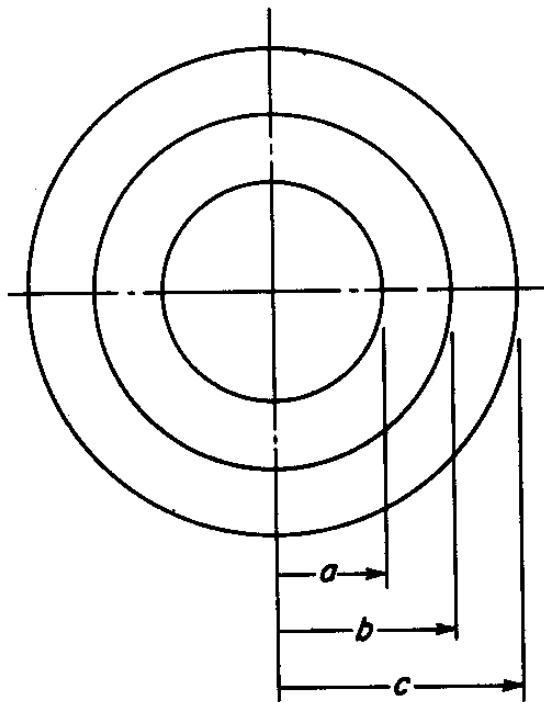


The interference pressure is that pressure needed to compress the inner cylinder and expand the outer cylinder so that the two cylinders can be assembled.

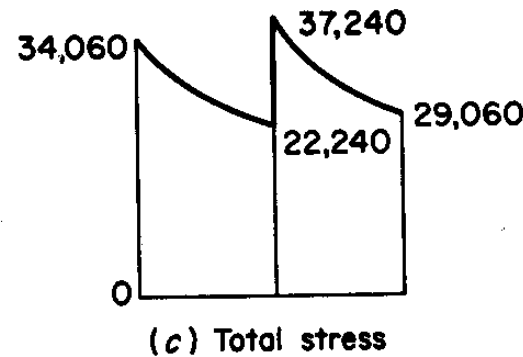
# Compound Cylinder (Shrink Fit Stresses)



**Hoop stress due to interference pressure**



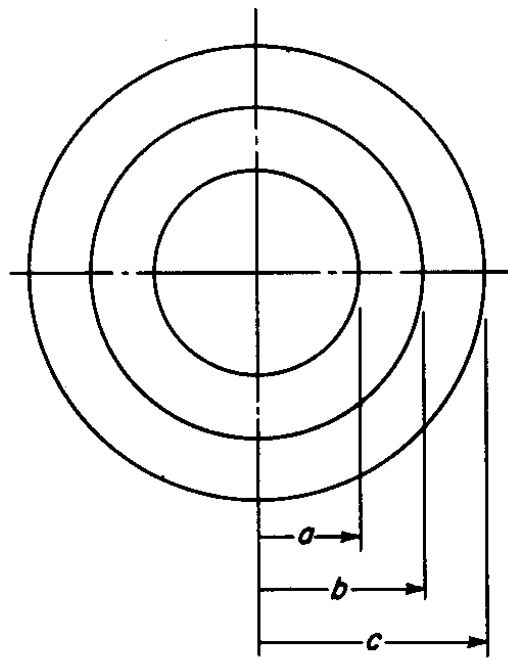
**Hoop stress due to an internal pressure**



**Total Hoop Stress**

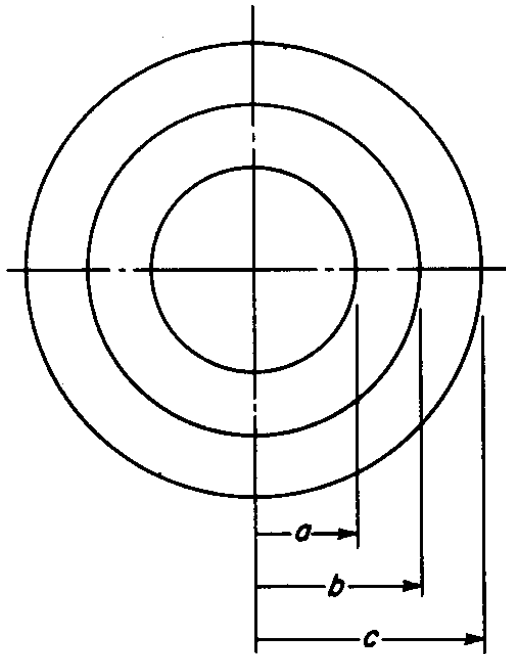
## Example Problem

Determine the tangential (hoop) stresses at the inner, outer, and mating surfaces of a compound cylinder subjected to an internal pressure of 20,000 psi. The radii are:  $a=6$  in,  $b=8$  in, and  $c=10$  in. The material is steel with a modulus of elasticity of  $E=30 \times 10^6$  psi, and the interference is 0.004 in.



# Example Problem

## (Interference Pressure)



$$p = \frac{E\delta}{b} \frac{(b^2 - a^2)(c^2 - b^2)}{2b^2(c^2 - a^2)}$$

$$p = 1,850 \text{ psi}$$

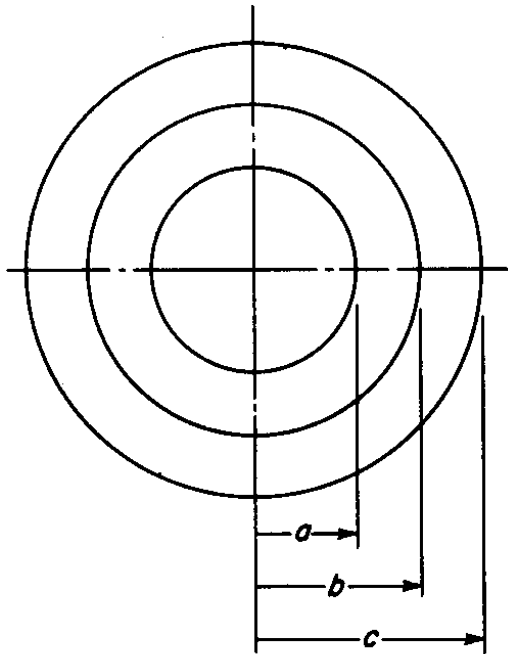
$$a = 6 \text{ in} \quad E = 30 \times 10^6 \text{ psi}$$

$$b = 8 \text{ in} \quad \delta = 0.004 \text{ in}$$

$$c = 10 \text{ in}$$

# Example Problem

## (Inner Cylinder)



$$\begin{aligned} a &= 6 \text{ in} & E &= 30 \times 10^6 \text{ psi} \\ b &= 8 \text{ in} & \delta &= 0.004 \text{ in} \\ c &= 10 \text{ in} & p &= 1,850 \text{ psi} \end{aligned}$$

### Lame' Equation

$$\sigma_{\theta} = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{(p_i - p_o) a^2 b^2}{(b^2 - a^2) r^2}$$

### External Pressure

$$\sigma_{\theta} = \frac{-b^2 p}{b^2 - a^2} + \frac{-p a^2 b^2}{(b^2 - a^2) r^2}$$

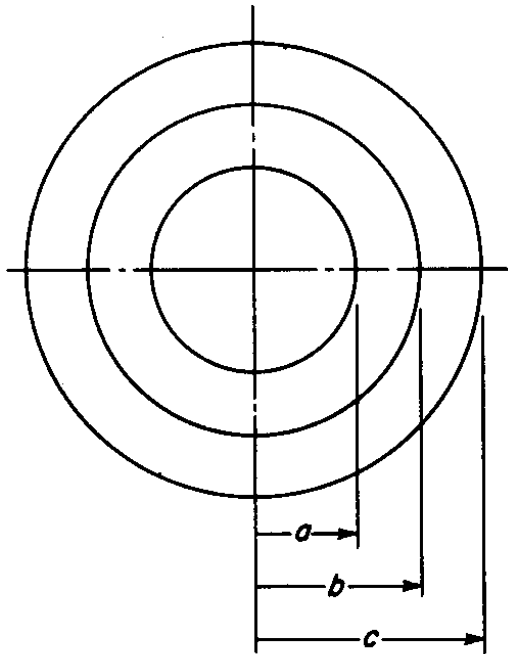
### Interference Stresses

$$\sigma_{\theta}|_{r=a} = \frac{-2pb^2}{b^2 - a^2} = -8,440 \text{ psi}$$

$$\sigma_{\theta}|_{r=b} = \frac{-2p(b^2 + a^2)}{b^2 - a^2} = -6,590 \text{ psi}$$

# Example Problem

## (Outer Cylinder)



$a=6$  in       $E = 30 \times 10^6$  psi  
 $b=8$  in       $\delta = 0.004$  in  
 $c=10$  in      $p = 1,850$  psi

### Lame' Equation

$$\sigma_{\theta} = \frac{b^2 p_i - c^2 p_o}{c^2 - b^2} + \frac{(p_i - p_o) b^2 c^2}{(c^2 - b^2) r^2}$$

### External Pressure

$$\sigma_{\theta} = \frac{b^2 p}{c^2 - b^2} + \frac{p b^2 c^2}{(c^2 - b^2) r^2}$$

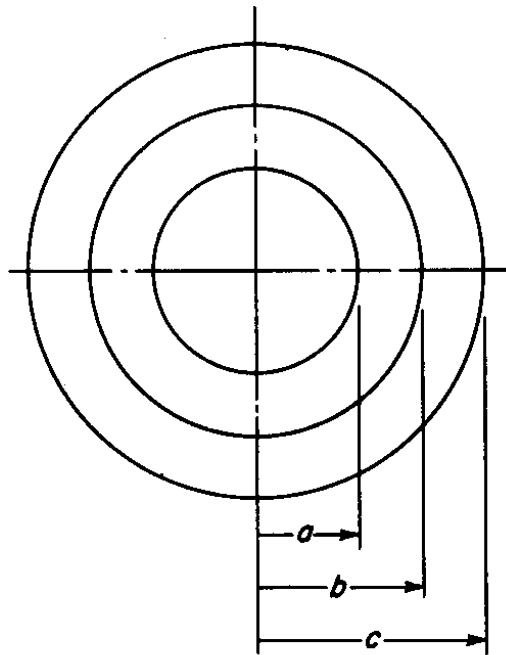
### Interference Stresses

$$\sigma_{\theta}|_{r=b} = p \left( \frac{b^2 + c^2}{c^2 - b^2} \right) = 8,410 \text{ psi}$$

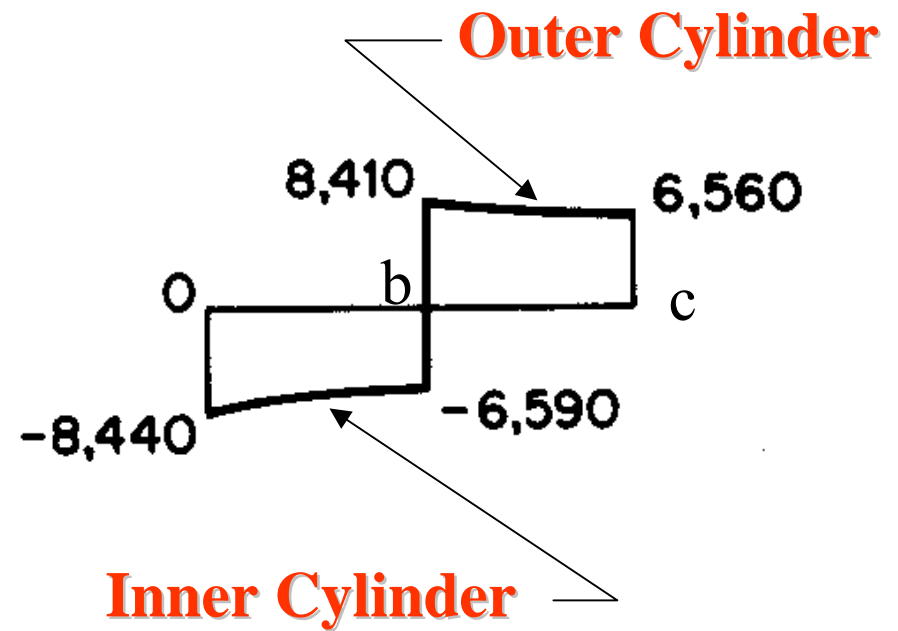
$$\sigma_{\theta}|_{r=c} = \frac{2pb^2}{c^2 - b^2} = 6,560 \text{ psi}$$

# Example Problem

## (Shrink-fit Stress Distribution)



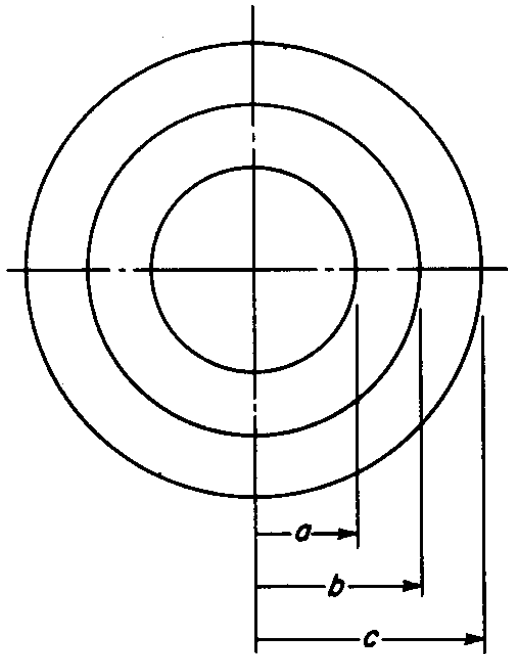
$$\begin{aligned}
 a &= 6 \text{ in} & E &= 30 \times 10^6 \text{ psi} \\
 b &= 8 \text{ in} & \delta &= 0.004 \text{ in} \\
 c &= 10 \text{ in} & p &= 1,850 \text{ psi}
 \end{aligned}$$



Harvey, Fig. 2.23(a)

# Example Problem

## (Internal Pressure)



$$\begin{aligned} a &= 6 \text{ in} & E &= 30 \times 10^6 \text{ psi} \\ b &= 8 \text{ in} & \delta &= 0.004 \text{ in} \\ c &= 10 \text{ in} & p_i &= 20,000 \text{ psi} \end{aligned}$$

### Lame' Equation

$$\sigma_{\theta} = \frac{a^2 p_i - c^2 p_o}{c^2 - a^2} + \frac{(p_i - p_o) a^2 c^2}{(c^2 - a^2) r^2}$$

### Internal Pressure

$$\sigma_{\theta} = \frac{a^2 p_i}{c^2 - a^2} + \frac{p_i a^2 c^2}{(c^2 - a^2) r^2}$$

### Radial Locations

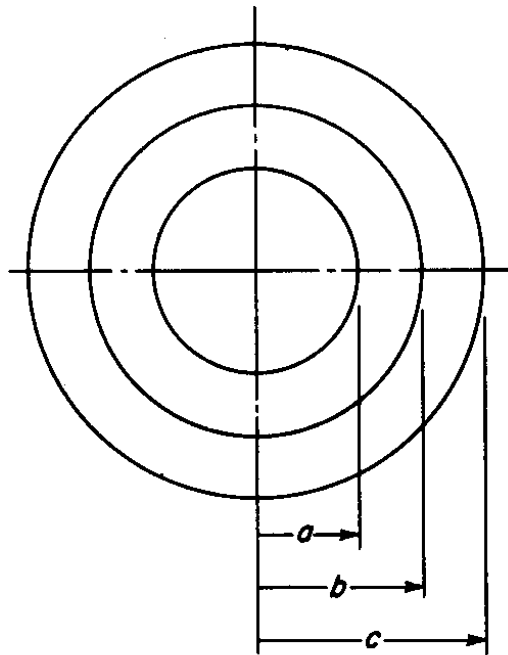
$$\sigma_{\theta} \Big|_{r=a} = 42,500 \text{ psi}$$

$$\sigma_{\theta} \Big|_{r=b} = 28,830 \text{ psi}$$

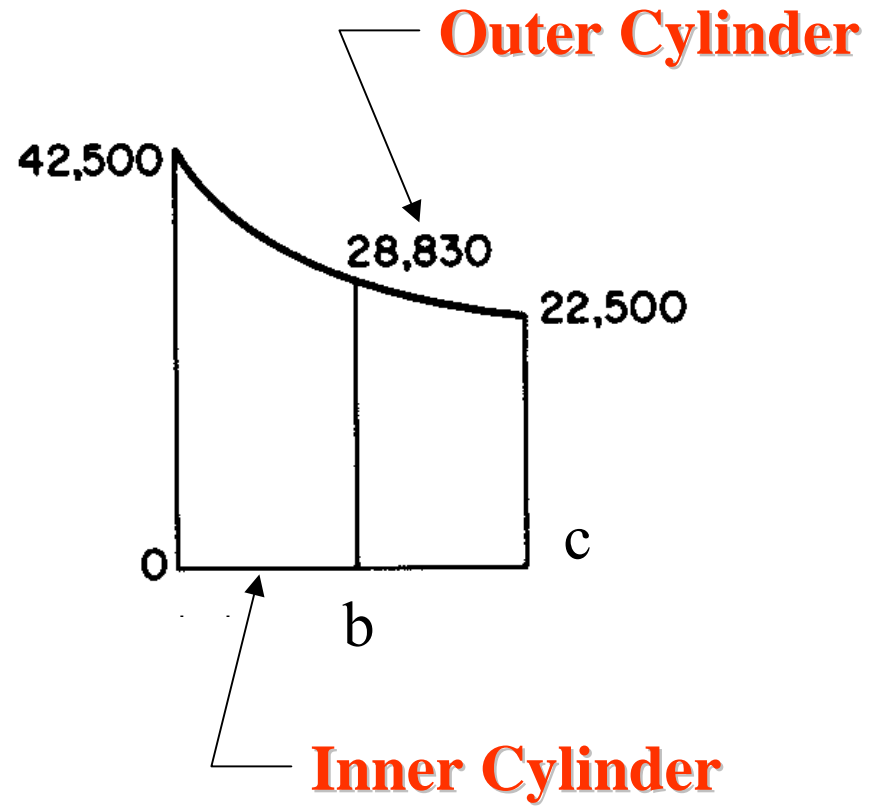
$$\sigma_{\theta} \Big|_{r=c} = 22,500 \text{ psi}$$

# Example Problem

## (Internal Pressure Stress Distribution)

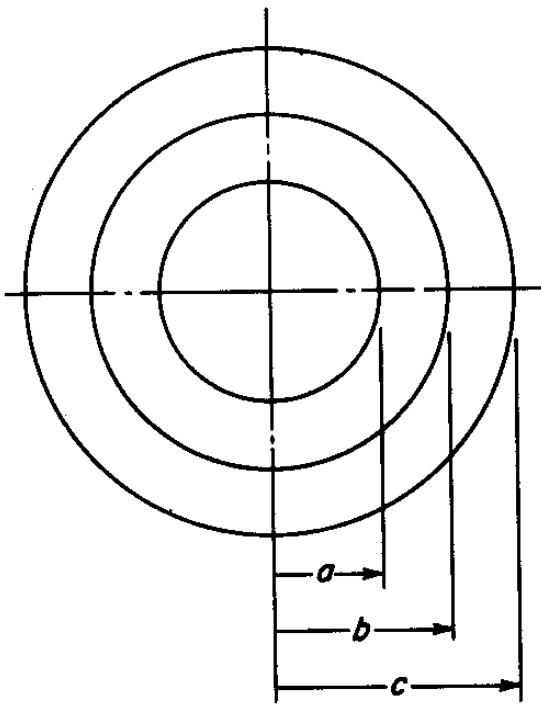
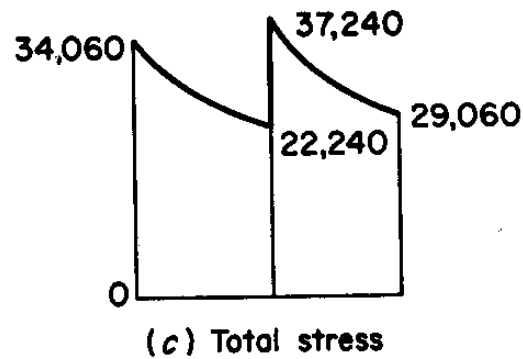
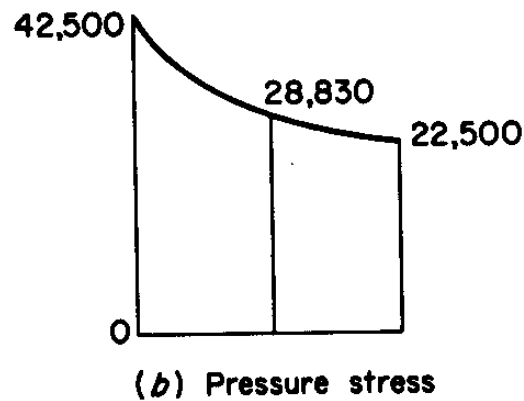
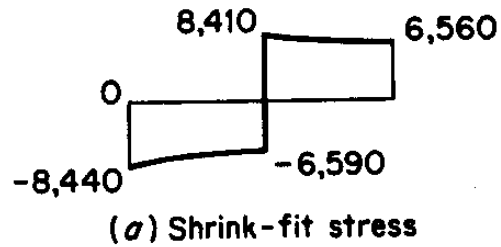


$$\begin{aligned} a &= 6 \text{ in} & E &= 30 \times 10^6 \text{ psi} \\ b &= 8 \text{ in} & \delta &= 0.004 \text{ in} \\ c &= 10 \text{ in} & p_i &= 20,000 \text{ psi} \end{aligned}$$



# Example Problem

## (Total Stresses)

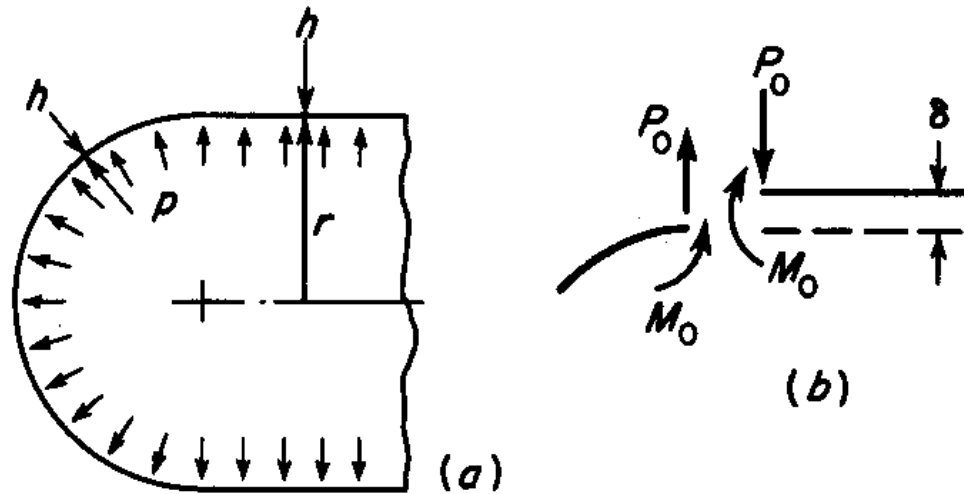


Note that the total stress on the inside of the cylinder is approximately 20% less than what it would have been without the shrink-fit stresses.

# Discontinuity Stresses

- The stresses in thick and thin walled cylinders (pressure vessels) considered so far have considered only the cylinder.
- There are often high stresses at geometric discontinuities in cylinders.
- These high stresses are similar to stress concentrations.

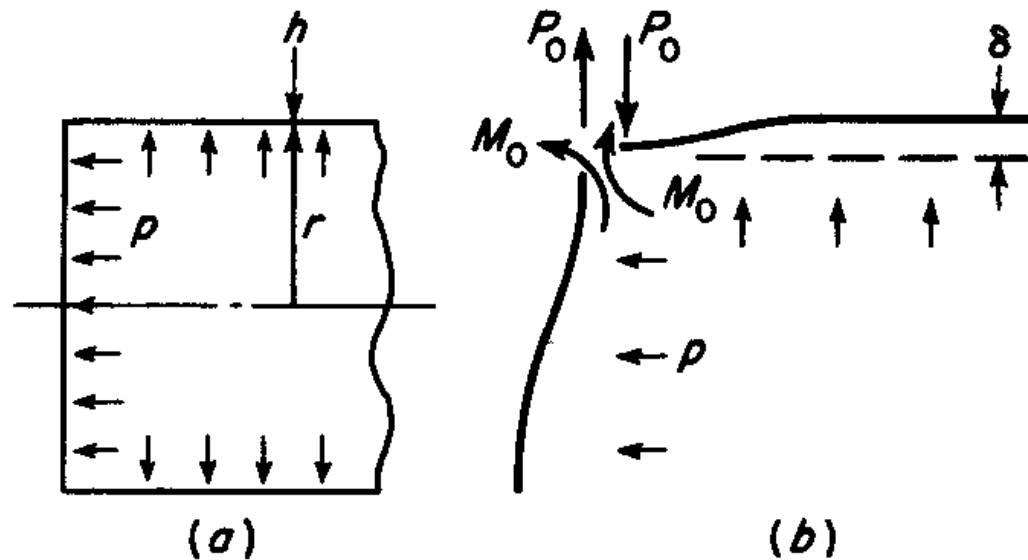
# Discontinuity Stresses (Example)



## Discontinuity at Hemispherical Head and Cylindrical Shell Junction

The force and moment required to cause the head and shell to displace and rotate the same amount will cause local bending stresses in both the head and shell.

# Discontinuity Stresses (Example)



## Discontinuity at Flat Head and Cylindrical Shell Junction

Hand stress analysis or finite element methods may be used to accurately compute the stresses around geometric discontinuities.

# Assignment

1. What is the required thickness of a 6 ft inside diameter cylinder, considering it as a thin wall vessel, to withstand an internal pressure of 1,000 psi if the allowable tangential stress is 20,000 psi.
2. A cylinder with a 48 in inside diameter and a 60 in outside diameter is subjected to an internal pressure of 5,000 psi. Determine value and place of occurrence the maximum tangential stress, the maximum radial stress, and the maximum shear stress.



# Assignment (Continued)



3. Determine the tangential (hoop) stresses at the inside radius of a compound cylinder subjected to an internal pressure of 32,000 psi. The radii are:  $a=10$  in,  $b=12$  in, and  $c=13$  in. The material is steel with a modulus of elasticity of  $E=30 \times 10^6$  psi, and the interference is 0.005 in.