The University of Tennessee at Martin

MARTIN School of Engineering

Hydrodynamic Bearings - Theory

Lecture 25

Engineering 473
Machine Design

Lubrication Zones

Boundary Lubrication

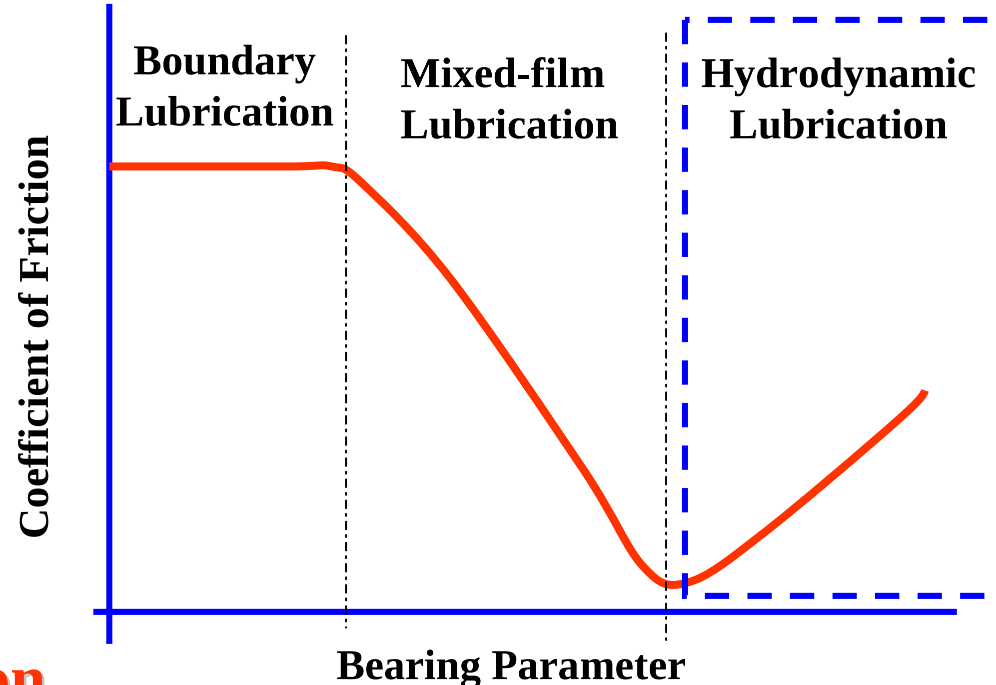
Contact between journal and bearing

Mixed-film Lubrication

Intermittent contact

Hydrodynamic Lubrication

Journal rides on a fluid film. Film is created by the motion of the journal.



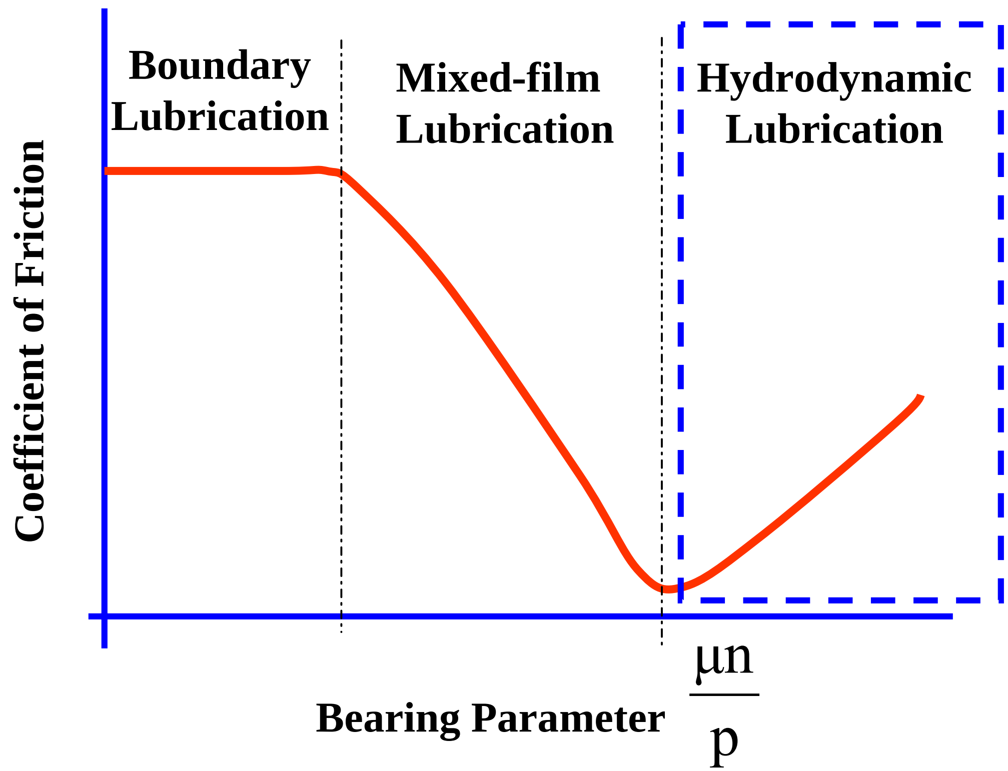
$$\text{Bearing Parameter} \equiv \frac{\mu n}{p}$$

$\mu \equiv$ dynamic viscosity, lb - sec/in^2

$n \equiv$ rotational speed, rev/sec

$p \equiv$ pressure (force/projected area), psi

Stable/Unstable Lubrication



Hydrodynamic Lubrication is often referred to as stable lubrication.

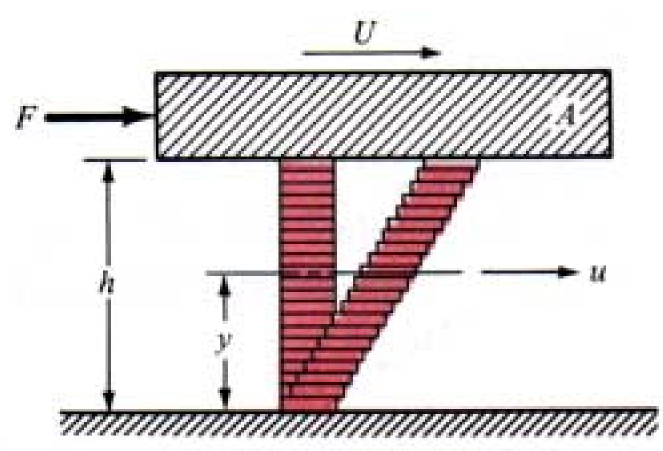
If the lubrication temperature increases, the viscosity drops. This results in a lower coefficient of friction, that causes the lubrication temperature to drop. => Self Correcting.

Mixed-film lubrication is unstable – an increase in lubrication temperature causes further increases in lubrication temperature.

Newtonian Fluid

A Newtonian fluid is any fluid whose shear stress and transverse rate of deformation are related through the equation.

$$\tau = \mu \frac{du}{dy}$$



Dynamic Viscosity

$$\mu = \tau \Big/ \frac{du}{dy}$$

Units

ips

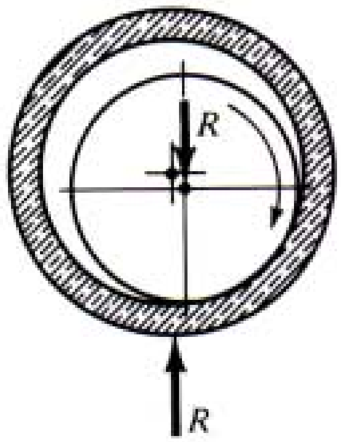
$$\frac{lb_f}{in^2} \Big/ \frac{\frac{in}{sec}}{in} = \frac{lb_f - sec}{in^2} = reyn$$

SI

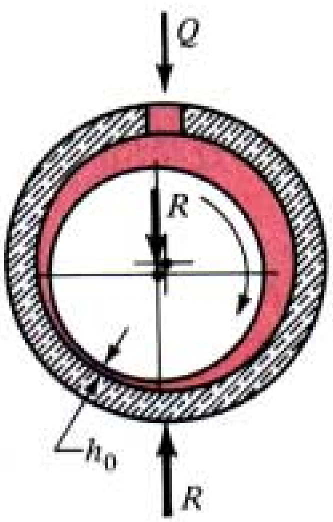
$$\frac{N}{m^2} \Big/ \frac{\frac{m}{sec}}{m} = \frac{N - sec}{m^2}$$

Other common units are discussed in the text.

Pumping Action



(a) Dry

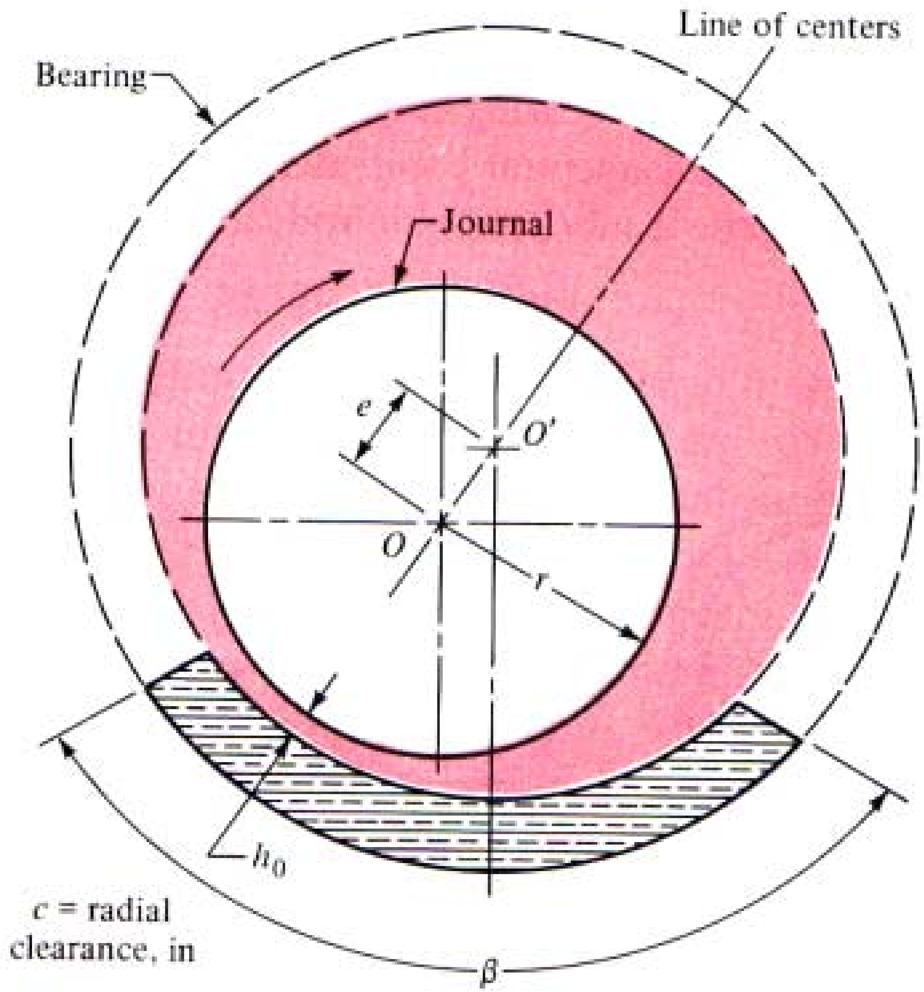


(b) Lubricated

When dry, friction will cause the journal to try to climb bearing inner wall.

When lubricant is introduced, the "climbing action" and the viscosity of the fluid will cause lubricant to be drawn around the journal creating a film between the journal and bearing. The lubricant pressure will push the journal to the side.

Journal Bearing Nomenclature



β is equal to 2π for a full bearing

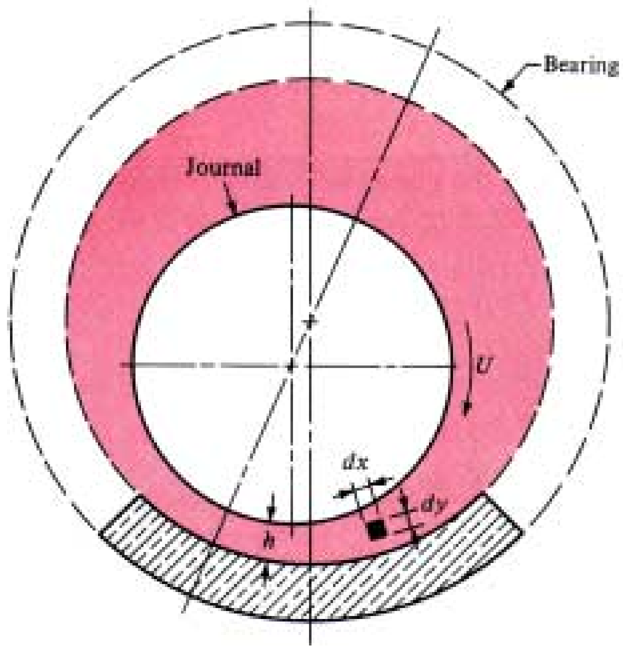
If β is less than 2π, it is known as a partial bearing.

We will only be considering the full bearing case.

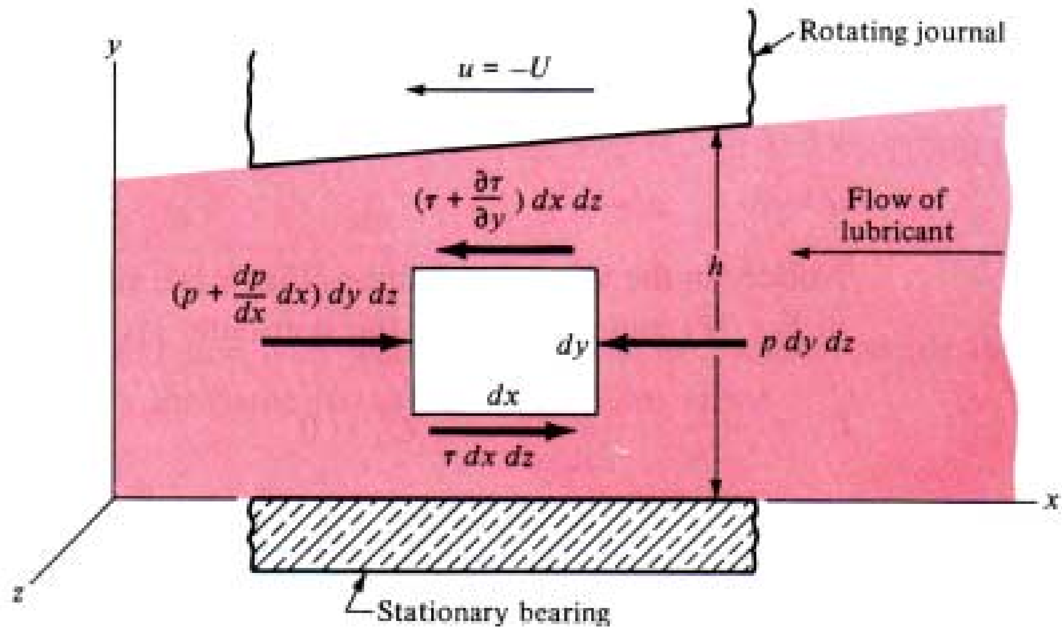
Analysis Assumptions

1. Lubricant is a Newtonian fluid
2. Inertia forces of the lubricant are negligible
3. Incompressible
4. Constant viscosity
5. Zero pressure gradient along the length of the bearing
6. The radius of the journal is large compared to the film thickness

Analysis Geometry



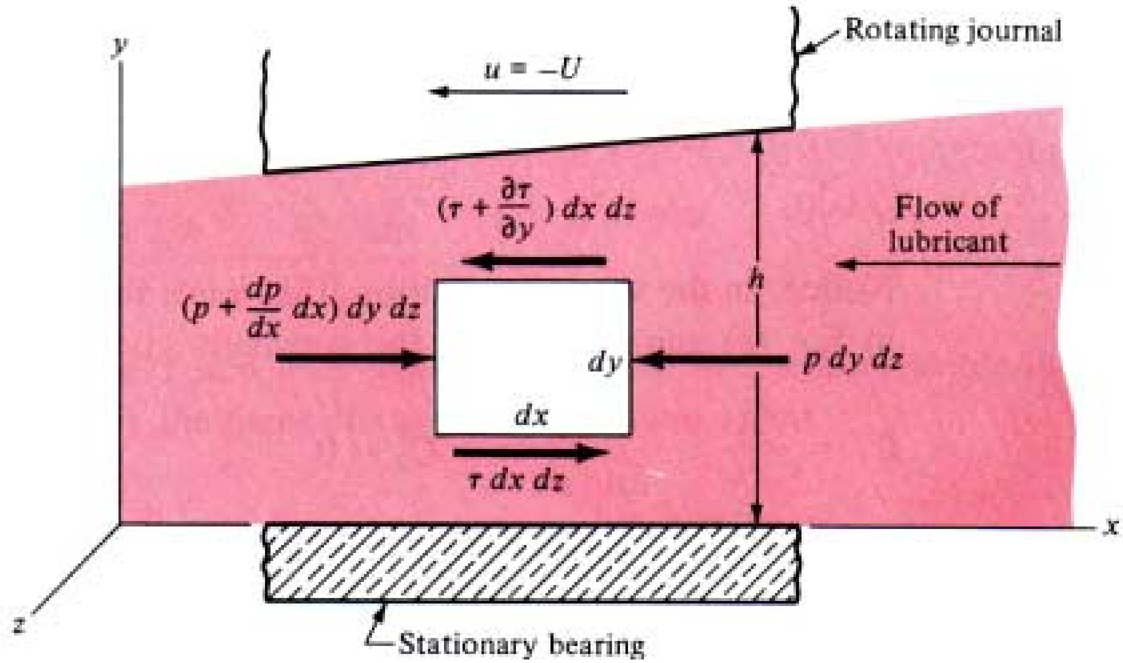
Actual Geometry



Unrolled Geometry

From boundary layer theory, the pressure gradient in the y direction is constant.

X-Momentum Equation



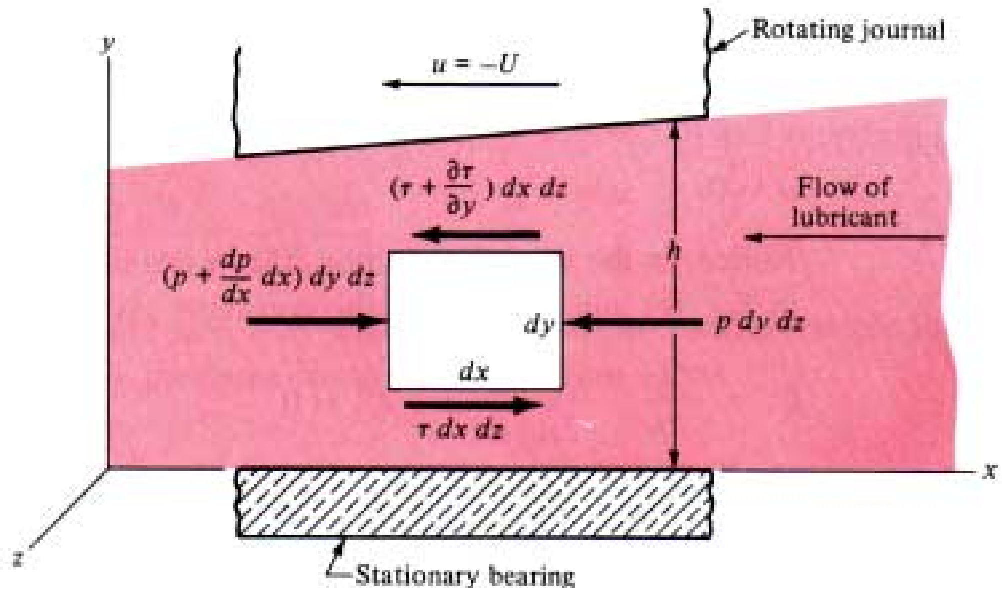
$$\sum F_x = 0 = \left(p + \frac{dp}{dx}dx\right)dydz + \tau dxdz - \left(\tau + \frac{\partial \tau}{\partial y}dy\right)dxdz - pdydz$$

$$\frac{dp}{dx} = \frac{\partial \tau}{\partial y} \quad \Rightarrow \quad \tau = \mu \frac{\partial u}{\partial y} \quad \Rightarrow \quad \boxed{\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2}}$$

X-Momentum Equation
(Continued)

X-Momentum Eq.

$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2}$$



General Solution

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{dp}{dx}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{dp}{dx}y + C_1(x)$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + C_1(x)y + C_2(x)$$

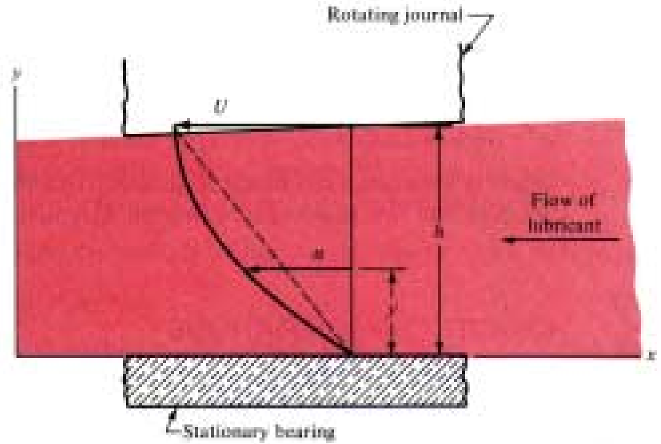
Boundary Conditions

$$y = 0,\ \ u = 0$$

$$y = h(x),\ \ u = \text{-}U$$

X-Momentum Equation
(Continued)

$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + C_1(x)y + C_2(x)$$



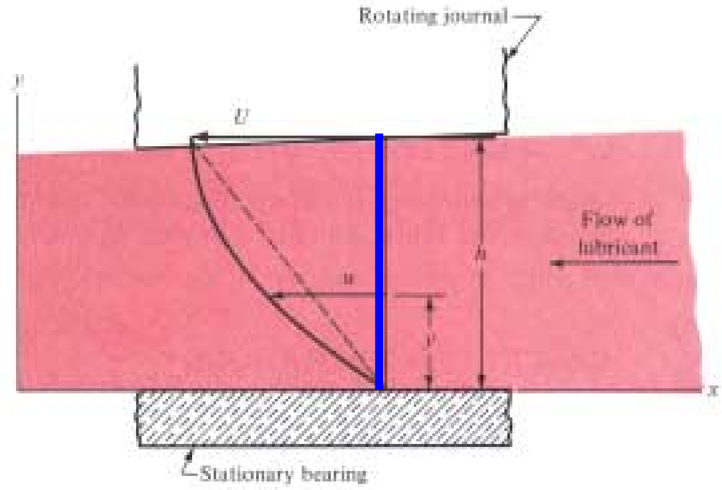
$$y = 0,\ u = 0 \quad \Rightarrow \quad C_2(x) = 0$$

$$y = h(x),\ u = \text{-}U \quad \Rightarrow \quad C_1(x) = -\frac{U}{h(x)} - \frac{h(x)}{2\mu}\frac{dp}{dx}$$

$$\boxed{u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - h(x)y\right) - \frac{U}{h(x)}y}$$

Note that h(x) and dp/dx are not known at this point.

Mass Flow Rate



$$\dot{m} = \rho \int_0^{h(x)} u\,dy$$

$$\dot{m} = \rho \int_0^{h(x)} \left(\frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - h(x)y\right) - \frac{U}{h(x)} y \right) dy$$

$$\dot{m} = \rho \left[-\frac{h(x)^3}{12\mu}\frac{dp}{dx} - \frac{Uh(x)}{2} \right]$$

Conservation of Mass

$$\dot{m} = \rho\left[-\frac{h(x)^3}{12\mu}\frac{dp}{dx} - \frac{Uh(x)}{2}\right]$$

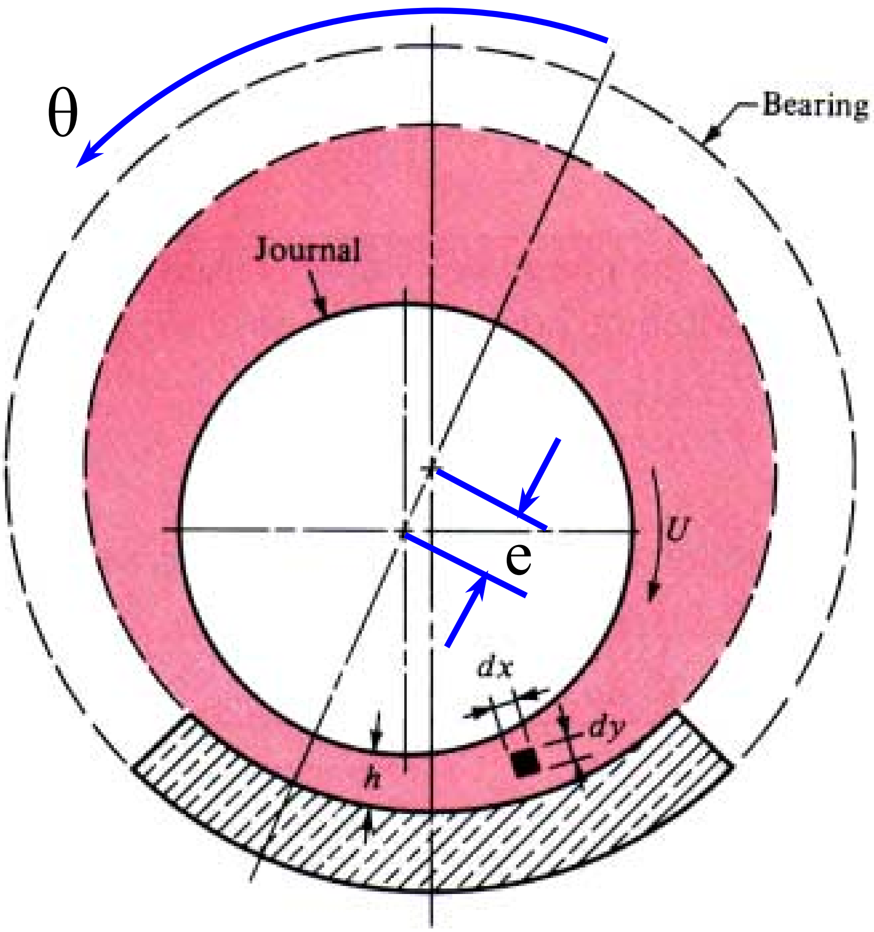
Conservation of Mass Requires

$$\frac{d\dot{m}}{dx} = 0 \quad \Rightarrow \quad -\frac{d}{dx}\left(\frac{h(x)^3}{12\mu}\frac{dp}{dx}\right) - \frac{U}{2}\frac{dh}{dx} = 0$$

$$\Rightarrow \quad \boxed{\frac{d}{dx}\left(\frac{h(x)^3}{\mu}\frac{dp}{dx}\right) = -6U\frac{dh}{dx}}$$

Reynold's Equation

h(x) Relationship



c_r = radial clearance

$$\varepsilon = \frac{e}{c_r}$$

$$h(\theta) = c_r(1 + \varepsilon \cdot \cos\theta)$$

$$h_{min} = c_r(1 - \varepsilon)$$

$$h_{max} = c_r(1 + \varepsilon)$$

$$h(x) = c_r\left(1 + \varepsilon \cdot \cos\left(\frac{2x}{D}\right)\right)$$

Sommerfeld Solution

$$\frac{d}{dx}\left(\frac{h(x)^3}{\mu}\frac{dp}{dx}\right) = -6U\frac{dh}{dx}$$

A. Sommerfeld solved these equations in 1904 to find the pressure distribution around the bearing.

$$h(x) = c_r\left(1 + \varepsilon \cdot \cos\left(\frac{2x}{D}\right)\right)$$

It is known as a “long bearing” solution because there is no flow in the axial direction.

$$p = \frac{\mu U r}{c_r^2}\left[\frac{6\varepsilon \cdot \sin\theta \cdot (2 + \varepsilon\cos\theta)}{(2+\varepsilon^2)(1+\varepsilon\cos\theta)^2}\right] + p_o \quad 0 \le \theta \le \pi$$

r is the journal radius, ε is a chosen design parameter.

Ocvirk Short-Bearing Solution

A "short bearing" allows lubricant flow in the longitudinal direction, z, as well as in the circumferential direction, x.

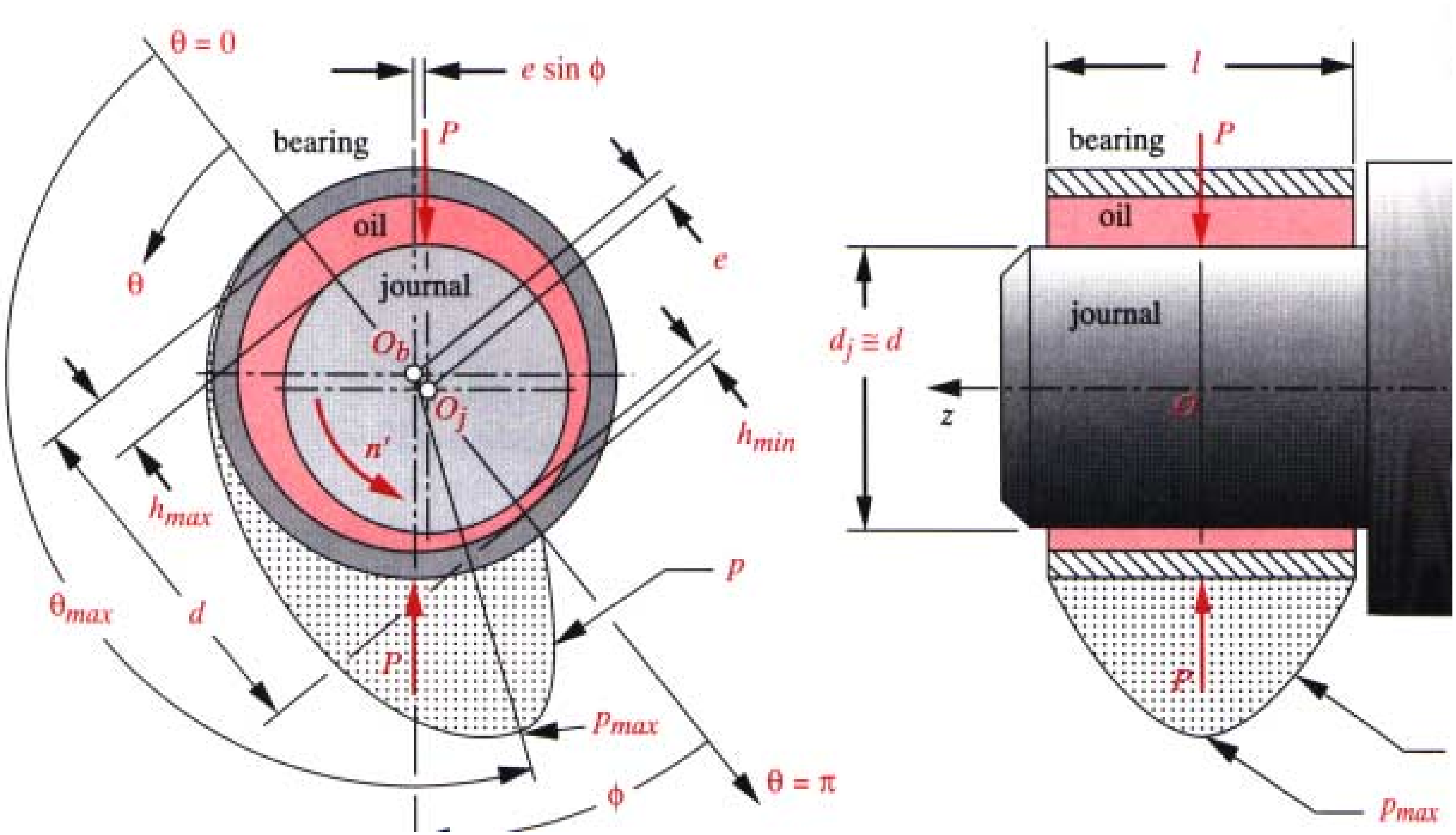
$$\frac{\partial}{\partial x}\left(\frac{h(x)^3}{\mu}\frac{dp}{dx}\right)-\frac{\partial}{\partial z}\left(\frac{h(x)^3}{\mu}\frac{dp}{dz}\right)=-6U\frac{\partial h}{\partial x}$$

Governing Equation

The Ocvirk solution (1955) neglects the first term as being small compared to the axial flow.

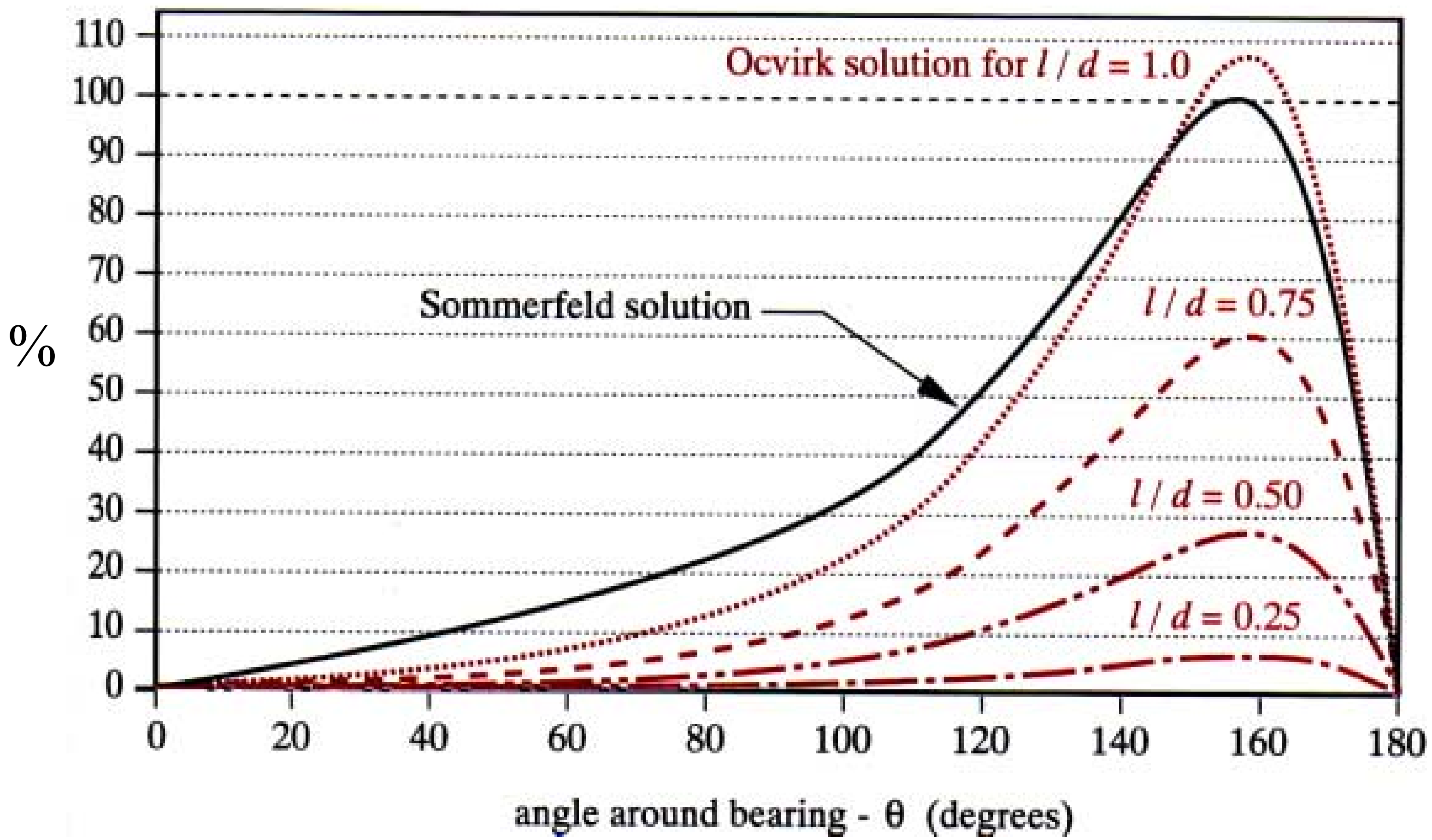
$$p=\frac{\mu U}{rc_r^2}\left(\frac{l^2}{4}-z^2\right)\frac{3\varepsilon\cdot\sin\theta}{(1+\varepsilon\cdot\cos\theta)^3}\qquad 0\le\theta\le\pi$$

Short-Bearing Pressure Distributions



Norton Fig. 10-8 & 10-9

Short & Long Bearing Comparisons



Assignment

Use Matlab to plot the pressure distribution predicted by the Sommerfeld equation for a journal bearing having a clearance ratio of 0.0017, journal radius of 0.75 in, ε of 0.6, μ=2.2μreyn, shaft rotational speed=20 rev/sec, and p_o=o.

First, generate the plot only for the range θ equals 0 to π.

Second, generate the plot for the range θ equals 0 to 2π. What happens to the pressure distribution from π to 2π. Is this physically possible? Discuss what would happen to the lubricant if this pressure distribution occurred.