The University of Tennessee at Martin
MARTIN School of Engineering

Steady Load Failure Theories

Lecture 5

Engineering 473
Machine Design

Steady Load Failure Theories

Ductile Materials	• Maximum-Normal-Stress
	• Maximum-Normal-Strain
	• Maximum-Shear-Stress
	• Distortion-Energy
	  • Shear-Energy
	  • Von Mises-Hencky
	  • Octahedral-Shear-Stress
	• Internal-Friction
	• Fracture Mechanics
		Brittle Materials

Uniaxial Stress/Strain Field		Multiaxial Stress/Strain Field

Many theories have been put forth – some agree reasonably well with test data, some do not.

The Maximum-Normal-Stress Theory

Postulate: Failure occurs when one of the three principal stresses equals the strength.

σ_1, σ_2, and σ_3 are principal stresses

$$\sigma_1 > \sigma_2 > \sigma_3$$

Failure occurs when either

$\sigma_1 = S_t$ Tension

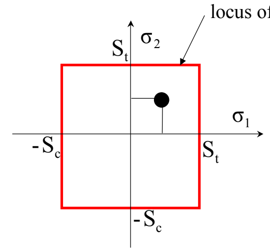
$\sigma_3 = -S_c$ Compression

$S_t \equiv$ Strength in Tension

$S_c \equiv$ Strength in Compression

Maximum-Normal-Stress Failure Surface

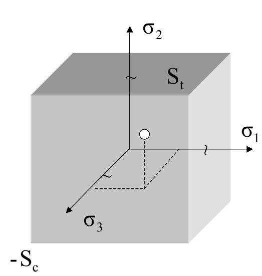
(Biaxial Condition)



According to the Maximum-Normal-Stress Theory, as long as stress state falls within the box, the material will not fail.

Maximum-Normal-Stress Failure Surface

(Three-dimensional Case)



According to the Maximum-Normal-Stress Theory, as long as stress state falls within the box, the material will not fail.

The Maximum-Normal-Strain Theory

(Saint-Venant's Theory)

Postulate: Yielding occurs when the largest of the three principal strains becomes equal to the strain corresponding to the yield strength.

$$E\varepsilon_1 = \sigma_1 - \nu(\sigma_2 + \sigma_3) = \pm S_y$$

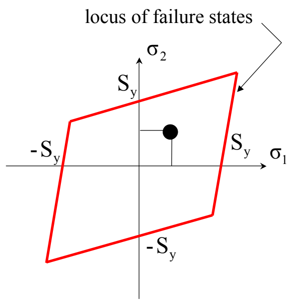
$$E\varepsilon_2 = \sigma_2 - \nu(\sigma_1 + \sigma_3) = \pm S_y$$

$$E\varepsilon_3 = \sigma_3 - \nu(\sigma_1 + \sigma_2) = \pm S_y$$

$E \equiv$ Young's Modulus

$\nu \equiv$ Poisson's Ratio

Maximum-Normal-Strain Theory
(Biaxial Condition)

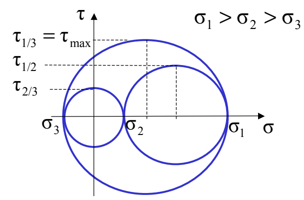


$$\sigma_1 - \nu\sigma_2 = \pm S_y$$
$$\sigma_2 - \nu\sigma_1 = \pm S_y$$

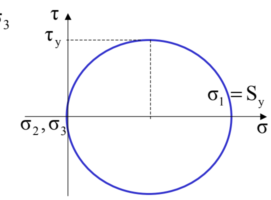
As long as the stress state falls within the polygon, the material will not yield.

Maximum-Shear-Stress Theory
(Tresca Criterion)

Postulate: Yielding begins whenever the maximum shear stress in a part becomes equal to the maximum shear stress in a tension test specimen that begins to yield.



Stress State in Part



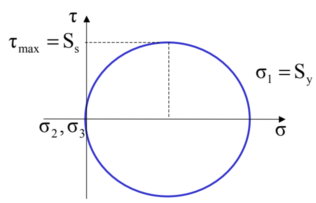
Tensile Test Specimen

Maximum-Shear-Stress Theory
(Continued)

Tensile Test Specimen

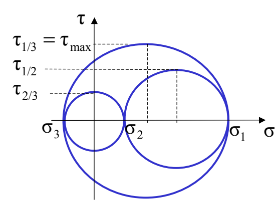
$$S_s = 0.5S_y$$

The shear yield strength is equal to one-half of the tension yield strength.



Maximum-Shear-Stress Theory
(Continued)

Stress State in Part



$\sigma_1 > \sigma_2 > \sigma_3$

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2}$$
$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2}$$
$$\tau_{1/3} = \tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

Maximum-Shear-Stress Theory
(Continued)

$$S_s = \frac{S_y}{2}$$

From Mohr's circle for a tensile test specimen

$$\tau_{1/3} = \tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

From Mohr's circle for a three-dimensional stress state.

$$S_y = \sigma_1 - \sigma_3$$

Maximum-Shear-Stress Theory
(Hydrostatic Effect)

Principal stresses **will always** have a hydrostatic component (equal pressure)

$$\sigma_1 = \sigma_1^d + \sigma^h$$
$$\sigma_2 = \sigma_2^d + \sigma^h$$
$$\sigma_3 = \sigma_3^d + \sigma^h$$

$$\sigma^h = \tfrac{1}{3} I_1 = \tfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

d => deviatoric component
h => hydrostatic

$$\tau_{1/2} = \frac{\sigma_1^d - \sigma_2^d}{2}$$
$$\tau_{2/3} = \frac{\sigma_2^d - \sigma_3^d}{2}$$
$$\tau_{1/3} = \frac{\sigma_1^d - \sigma_3^d}{2}$$

The maximum shear stresses are independent of the hydrostatic stress.

Maximum-Shear-Stress Theory

(Hydrostatic Effect – Continued)

Hydrostatic Stress State

If $\sigma_1^d = \sigma_2^d = \sigma_3^d$

Then $\tau_{max} = 0$, and there is no yielding regardless of the magintude of the hydrostatic stress.

The Maximum-Shear-Stress Theory postulates that yielding is independent of a hydrostatic stress.

Maximum-Shear-Stress Theory

(Biaxial Representation of the Yield Surface)

Yielding will occur if any of the following criteria are met.

$$\pm S_y = \sigma_1 - \sigma_2$$
$$\pm S_y = \sigma_2 - \sigma_3$$
$$\pm S_y = \sigma_1 - \sigma_3$$

For biaxial case (plane stress) $\sigma_3 = 0$

$$\pm S_y = \sigma_1 - \sigma_2$$
$$\pm S_y = \sigma_2$$
$$\pm S_y = \sigma_1$$

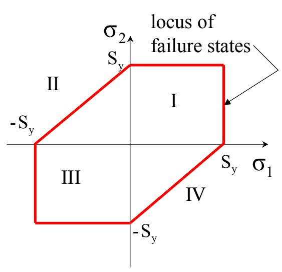
In general, all three conditions must be checked.

Maximum-Shear-Stress Theory

(Biaxial Representation of the Yield Surface)

For biaxial case (plane stress) $\sigma_3 = 0$

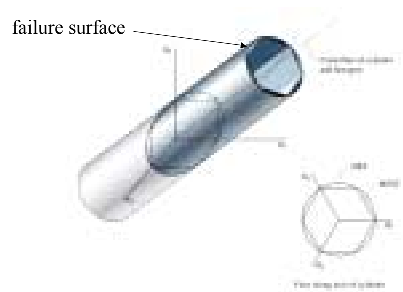
$$\pm S_y = \sigma_1 - \sigma_2$$
$$\pm S_y = \sigma_2$$
$$\pm S_y = \sigma_1$$



Note that in the I and III quadrants the Maximum-Shear-Stress Theory and Maximum-Normal-Stress Theory are the same for the biaxial case.

Maximum-Shear-Stress Theory

(Three-dimensional Representation of the Yield Surface)



Hamrock, Fig. 6.9

Assignment

Failure Theories, Read Section 5-9.

(a) Find the bending and transverse shear stress at points A and B in the figure. (b) Find the maximum normal stress and maximum shear stress at both points. (c) For a yield point of 50,000 psi, find the factor of safety based on the maximum normal stress theory and the maximum shear stress theory.