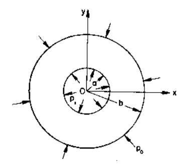
The University of Tennessee at Martin
MARTIN School of Engineering

Compound Cylinders & Discontinuity Stresses

Lecture 14

Engineering 473
Machine Design

Lame' Equations for Thick Walled Cylinders

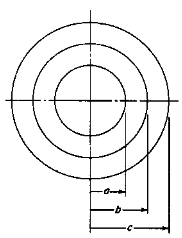


$$\sigma_r = \frac{a^2p_i - b^2p_o}{b^2 - a^2} - \frac{(p_i - p_o)a^2b^2}{(b^2 - a^2)r^2}$$

$$\sigma_\theta = \frac{a^2p_i - b^2p_o}{b^2 - a^2} + \frac{(p_i - p_o)a^2b^2}{(b^2 - a^2)r^2}$$

$$u = \frac{1-\nu}{E}\frac{(a^2p_i - b^2p_o)r}{b^2 - a^2} + \frac{1+\nu}{E}\frac{(p_i - p_o)a^2b^2}{(b^2 - a^2)r}$$

Compound Cylinders



Civil War Parrott Guns

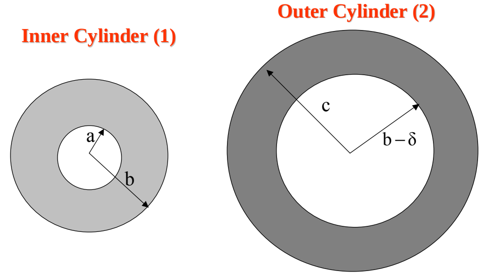
Compound cylinders are used to increase the pressure that can be contained in cylinders.

www.wwd.net/steen

Compound Cylinders
(Assembly)

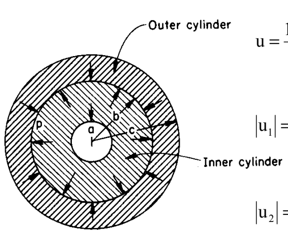
Inner Cylinder (1)

Outer Cylinder (2)



The inside diameter of cylinder 2 is undersized by a small amount (interference) and must be heated to fit over cylinder 1. This is often referred to as a shrink fit.

Compound Cylinders
(Interference Equations)



Lame's Equation

$$u = \frac{1-\nu}{E}\frac{(a^2p_i - b^2p_o)r}{b^2 - a^2} + \frac{1+\nu}{E}\frac{(p_i - p_o)a^2b^2}{(b^2 - a^2)r}$$

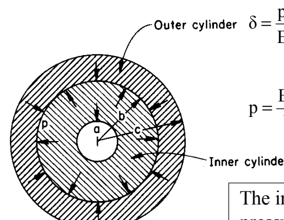
$$|u_1| = \frac{pb}{E_i}\left(\frac{b^2 + a^2}{b^2 - a^2} - \nu_i\right)$$ Inner Cylinder

$$|u_2| = \frac{pb}{E_o}\left(\frac{b^2 + c^2}{c^2 - b^2} + \nu_o\right)$$ Outer Cylinder

The interface pressure is directly proportional to the interference.

$$\delta = \frac{pb}{E_o}\left(\frac{b^2 + c^2}{c^2 - b^2} + \nu_o\right) + \frac{pb}{E_i}\left(\frac{b^2 + a^2}{b^2 - a^2} - \nu_i\right)$$

Compound Cylinders
(Interference Pressure)



$$\delta = \frac{pb}{E_o}\left(\frac{b^2 + c^2}{c^2 - b^2} + \nu_o\right) + \frac{pb}{E_i}\left(\frac{b^2 + a^2}{b^2 - a^2} - \nu_i\right)$$

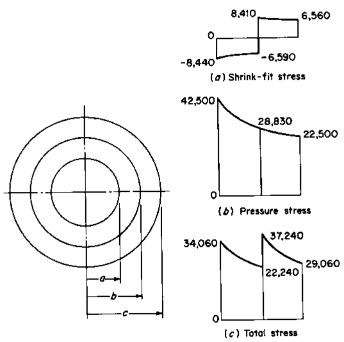
$$p = \frac{E\delta}{b}\frac{(b^2 - a^2)(c^2 - b^2)}{2b^2(c^2 - a^2)}$$ For same materials

The interference pressure is that pressure needed to compress the inner cylinder and expand the outer cylinder so that the two cylinders can be assembled.

Ugural, Fig. 8.5

Compound Cylinder

(Shrink Fit Stresses)



Hoop stress due to interference pressure

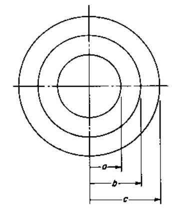
Hoop stress due to an internal pressure

Total Hoop Stress

Harvey, Fig. 2.23

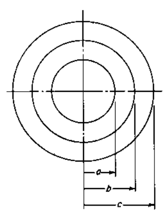
Example Problem

Determine the tangential (hoop) stresses at the inner, outer, and mating surfaces of a compound cylinder subjected to an internal pressure of 20,000 psi. The radii are: a=6 in, b=8 in, and c=10 in. The material is steel with a modulus of elasticity of E=30x10^6 psi, and the interference is 0.004 in.



Example Problem

(Interference Pressure)



$$p = \frac{E\delta}{b}\frac{(b^2-a^2)(c^2-b^2)}{2b^2(c^2-a^2)}$$

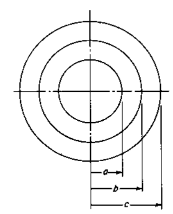
$$p = 1{,}850 \text{ psi}$$

a=6 in
b=8 in
c=10 in

$E = 30\text{x}10^6$ psi
$\delta = 0.004$ in

Example Problem

(Inner Cylinder)



Lame' Equation

$$\sigma_\theta = \frac{a^2p_i - b^2p_o}{b^2-a^2} + \frac{(p_i-p_o)a^2b^2}{(b^2-a^2)r^2}$$

External Pressure

$$\sigma_\theta = \frac{-b^2p}{b^2-a^2} + \frac{-pa^2b^2}{(b^2-a^2)r^2}$$

Interference Stresses

$$\sigma_\theta\big|_{r=a} = \frac{-2pb^2}{b^2-a^2} = -8{,}440 \text{ psi}$$

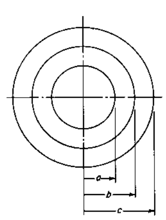
$$\sigma_\theta\big|_{r=b} = \frac{-2p(b^2+a^2)}{b^2-a^2} = -6{,}590\text{psi}$$

a=6 in
b=8 in
c=10 in

$E = 30\text{x}10^6$ psi
$\delta = 0.004$ in
$p = 1{,}850$ psi

Example Problem

(Outer Cylinder)



Lame' Equation

$$\sigma_\theta = \frac{b^2p_i - c^2p_o}{c^2-b^2} + \frac{(p_i-p_o)b^2c^2}{(c^2-b^2)r^2}$$

External Pressure

$$\sigma_\theta = \frac{b^2p}{c^2-b^2} + \frac{pb^2c^2}{(c^2-b^2)r^2}$$

Interference Stresses

$$\sigma_\theta\big|_{r=b} = p\left(\frac{b^2+c^2}{c^2-b^2}\right) = 8{,}410 \text{ psi}$$

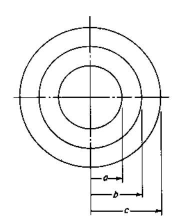
$$\sigma_\theta\big|_{r=c} = \frac{2pb^2}{c^2-b^2} = 6{,}560 \text{ psi}$$

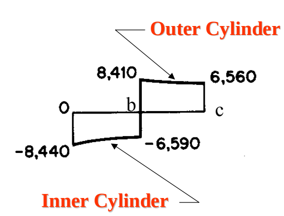
a=6 in
b=8 in
c=10 in

$E = 30\text{x}10^6$ psi
$\delta = 0.004$ in
$p = 1{,}850$ psi

Example Problem

(Shrink-fit Stress Distribution)



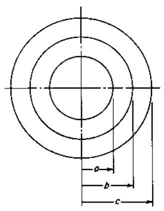


a=6 in
b=8 in
c=10 in

$E = 30\text{x}10^6$ psi
$\delta = 0.004$ in
$p = 1{,}850$ psi

Harvey, Fig. 2.23(a)

Example Problem
(Internal Pressure)



Lame' Equation

$$\sigma_\theta = \frac{a^2 p_i - c^2 p_o}{c^2 - a^2} + \frac{(p_i - p_o) a^2 c^2}{(c^2 - a^2) r^2}$$

Internal Pressure

$$\sigma_\theta = \frac{a^2 p_i}{c^2 - a^2} + \frac{p_i a^2 c^2}{(c^2 - a^2) r^2}$$

Radial Locations

$$\sigma_\theta \big|_{r=a} = 42{,}500 \text{ psi}$$

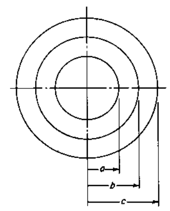
$$\sigma_\theta \big|_{r=b} = 28{,}830 \text{ psi}$$

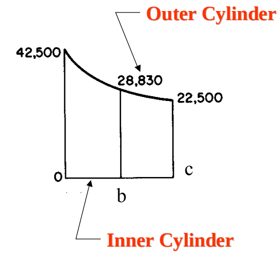
$$\sigma_\theta \big|_{r=c} = 22{,}500 \text{ psi}$$

a=6 in
b=8 in
c=10 in

$E = 30\text{x}10^6$ psi
$\delta = 0.004$ in
$p_i = 20{,}000$ psi

Example Problem
(Internal Pressure Stress Distribution)



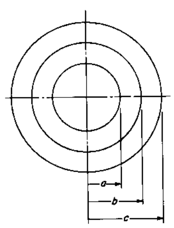


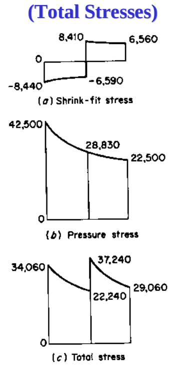
a=6 in
b=8 in
c=10 in

$E = 30\text{x}10^6$ psi
$\delta = 0.004$ in
$p_i = 20{,}000$ psi

Harvey, Fig. 2.23(b)

Example Problem
(Total Stresses)





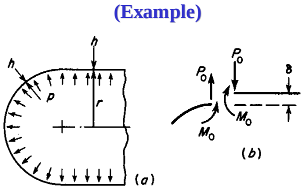
Note that the total stress on the inside of the cylinder is approximately 20% less than what it would have been without the shrink-fit stresses.

Harvey, Fig. 2.23

Discontinuity Stresses

- The stresses in thick and thin walled cylinders (pressure vessels) considered so far have considered only the cylinder.
- There are often high stresses at geometric discontinuities in cylinders.
- These high stresses are similar to stress concentrations.

Discontinuity Stresses
(Example)

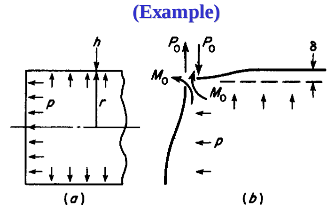


Discontinuity at Hemispherical Head and Cylindrical Shell Juncture

The force and moment required to cause the head and shell to displace and rotate the same amount will cause local bending stresses in both the head and shell.

Harvey, Fig. 4.8

Discontinuity Stresses
(Example)



Discontinuity at Flat Head and Cylindrical Shell Juncture

Hand stress analysis or finite element methods may be used to accurately compute the stresses around geometric discontinuities.

Assignment

1. What is the required thickness of a 6 ft inside diameter cylinder, considering it as a thin wall vessel, to withstand an internal pressure of 1,000 psi if the allowable tangential stress is 20,000 psi.
2. A cylinder with a 48 in inside diameter and a 60 in outside diameter is subjected to an internal pressure of 5,000 psi. Determine value and place of occurrence the maximum tangential stress, the maximum radial stress, and the maximum shear stress.

Assignment
(Continued)

3. Determine the tangential (hoop) stresses at the inside radius of a compound cylinder subjected to an internal pressure of 32,000 psi. The radii are: a=10 in, b=12 in, and c=13 in. The material is steel with a modulus of elasticity of $E=30x10^6$ psi, and the interference is 0.005 in.