

# Shaft Loading

## Lecture 17

Engineering 473  
 Machine Design



## Shaft Design Issues

**Shaft:** Rotating machine element that transmits power.

**Material**

$S_e$   $S_{ut}$   
 $K_{IC}$   $S_{yt}$   
 $R_c$   $q$

**Environment**

Temperature  
 Corrosion  
 Magnetic

**Tolerances**

**Loads**

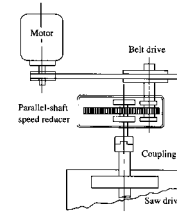
Stationary  
 Rotating

**Interfaces**

Press Fits  
 Splines  
 Bearings

**Assembly**

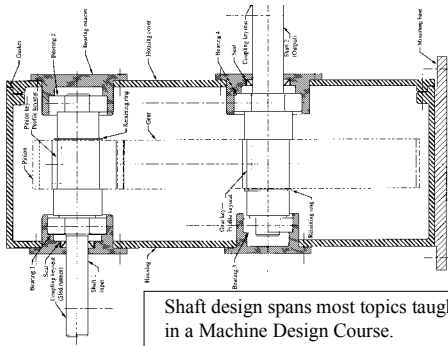
**Stiffness**



Shafts are one of the most commonly encountered machine components.

Mott, Fig. 5-1

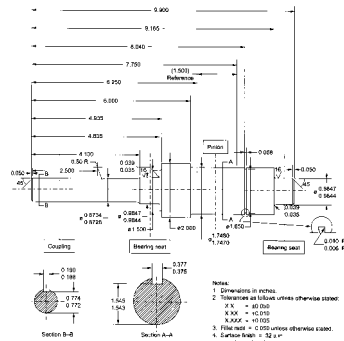
## Parallel Shaft Gear Box



Shaft design spans most topics taught in a Machine Design Course.

Mott, Fig. 15-7

## Design Detail Needed to Specify a Shaft



Significant detail is required to completely specify the geometry needed to fabricate a shaft.

Mott, Fig. 15-5

## Common Shaft Loading Mechanisms

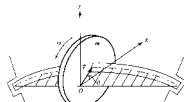
**Spur Gears**



**Chain Drives**



**Unbalanced Mass**



**Helical Gears**



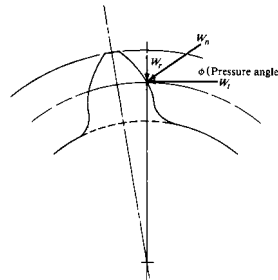
**Spiral Bevel Gears**



**Belt Drives**



## Spur Gear Loads



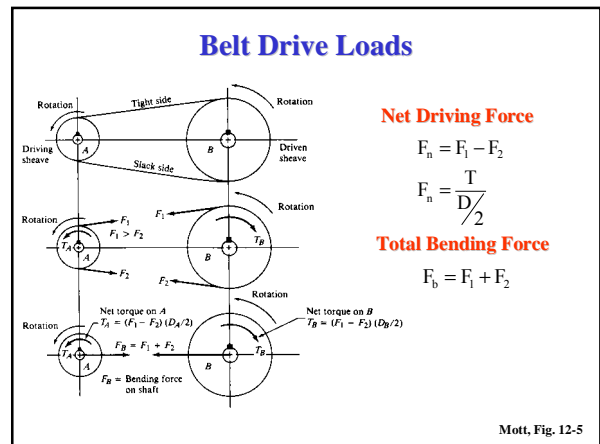
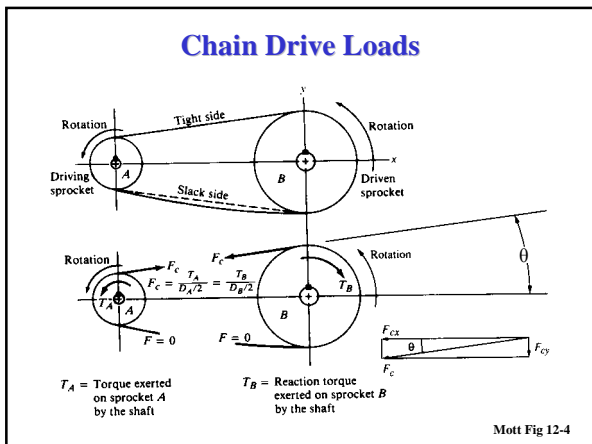
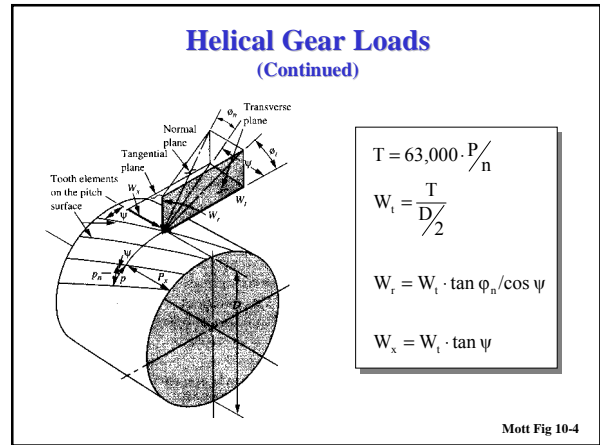
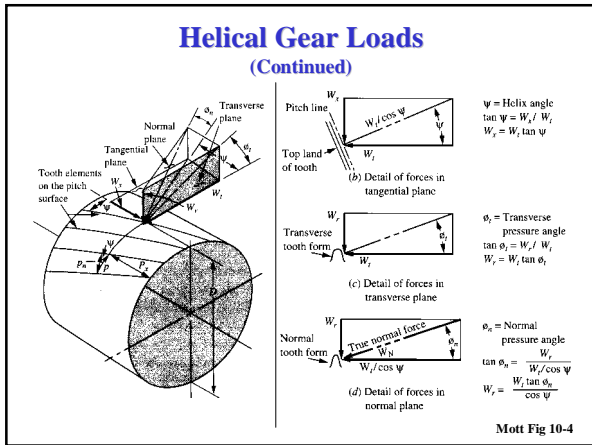
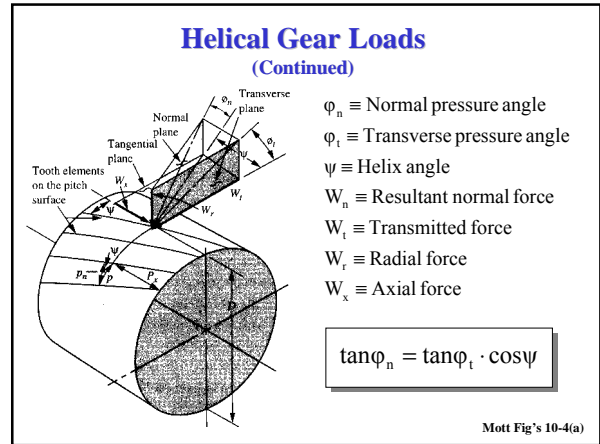
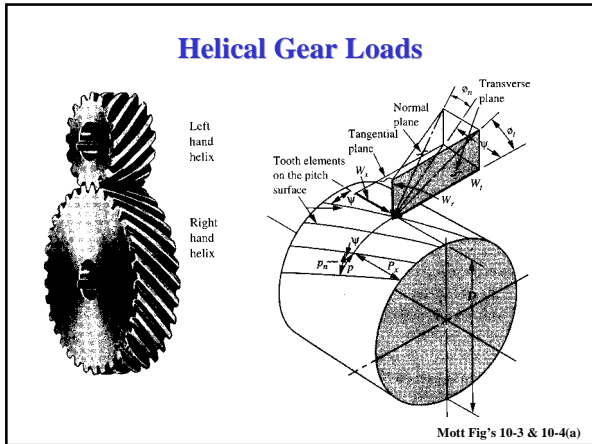
$$T = 63,000 \cdot \frac{P}{n}$$

$$W_t = \frac{T}{D/2}$$

$$W_r = W_t \cdot \tan \phi$$

$P$   $\equiv$  transmitted power [hp]  
 $n$   $\equiv$  rotational speed [rpm]  
 $T$   $\equiv$  shaft torque [in · lb]  
 $D$   $\equiv$  pitch diameter [in]  
 $\phi$   $\equiv$  pressure angle

Mott, Fig. 12-3



### Belt Drive Loads (Bending Force)

**Net Driving Force**

$$F_n = F_1 - F_2$$

$$C = \frac{F_b}{F_n} = \frac{F_1 + F_2}{F_1 - F_2}$$

$$F_n = \frac{T}{D/2}$$

**Total Bending Force**

$$F_b = F_1 + F_2$$

$$C = \frac{5F_2 + F_1}{5F_2 - F_1} = 1.5 \text{ (V-belts)}$$

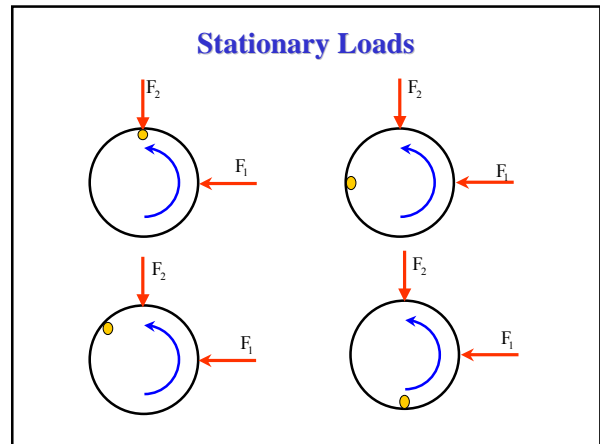
$$C = \frac{3F_2 + F_1}{3F_2 - F_1} = 2.0 \text{ (Flat-belts)}$$

**Tension Ratio**

$$\frac{F_1}{F_2} = 5.0 \text{ (V-belts)}$$

$$F_b = 1.5 F_n \text{ (V-belts)}$$

$$\frac{F_1}{F_2} = 3.0 \text{ (Flat-belts)}$$

$$F_b = 2.0 F_n \text{ (Flat-belts)}$$


### Bending Stresses Due to Stationary Loads

$$\sigma_b = \frac{M_2 c_3}{I_{22}} - \frac{M_3 c_2}{I_{33}}$$

$$\sigma_b = \frac{M_2 r \sin \theta}{I} - \frac{M_3 r \cos \theta}{I} \quad \text{Eq. 1}$$

$$\frac{\partial \sigma_b}{\partial \theta} = \frac{M_2 r \cos \theta}{I} + \frac{M_3 r \sin \theta}{I} = 0$$

$$\tan \theta = -\frac{M_2}{M_3} \quad \text{Eq. 2}$$

$I_{22} = I_{33} = I$   
 $c_2 = r \cos \theta$   
 $c_3 = r \sin \theta$

### Bending Stresses Due to Stationary Loads

$$M = \sqrt{M_2^2 + M_3^2} \quad \text{Eq. 3}$$

Combining with Eq. 2,

$$\tan \theta = \frac{-M_2/M}{M_3/M} = \frac{\sin \theta}{\cos \theta}$$

$$\sigma_b = \frac{M_2 r \sin \theta}{I} - \frac{M_3 r \cos \theta}{I} \quad \text{Eq. 1}$$

$$\tan \theta = -\frac{M_2}{M_3} \quad \text{Eq. 2}$$

$$\sin \theta = \frac{-M_2}{M} \quad \text{Eq. 4}$$

$$\cos \theta = \frac{M_3}{M}$$

### Bending Stresses Due to Stationary Loads

$$\sigma_b = \frac{M_2 r \sin \theta}{I} - \frac{M_3 r \cos \theta}{I} \quad \text{Eq. 1}$$

$$M = \sqrt{M_2^2 + M_3^2} \quad \text{Eq. 3}$$

$$\sin \theta = \frac{-M_2}{M}$$

$$\cos \theta = \frac{M_3}{M} \quad \text{Eq's 4}$$

Combining Eq's 1, 3, and 4

$$\sigma_b = \frac{M_2 r \sin \theta}{I} - \frac{M_3 r \cos \theta}{I}$$

$$\sigma_b = -\frac{M_2^2 r}{I} - \frac{M_3^2 r}{I}$$

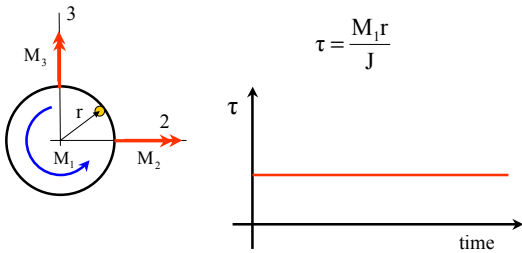
$$\sigma_b = -\frac{\sqrt{M_2^2 + M_3^2} \cdot r}{I}$$

### Bending Stresses Due to Stationary Loads

$$\sigma_{b,max} = \frac{\sqrt{M_2^2 + M_3^2} \cdot r}{I} \quad \sigma_{b,min} = -\frac{\sqrt{M_2^2 + M_3^2} \cdot r}{I}$$

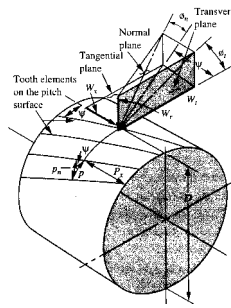
Mott, Fig. 5-3(e)

### Torsional Stresses Due to Stationary Loads



The torsional stress at a point will be constant under steady state conditions.

### Axial Stresses Due to Stationary Loads

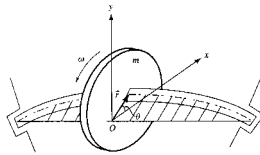


Helical, worm, and spiral gears will generate axial loads in the shaft. Under steady state conditions, the axial stress from these loads will be constant.

$$\sigma_x = \frac{W_x}{A}$$

Mott Fig 10-4

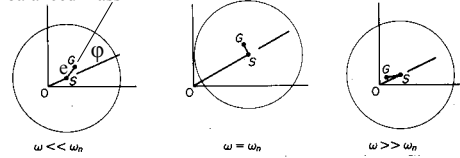
### Unbalanced Mass Loads



Bending stresses in a shaft due to in-balance loads are complicated by whether the rotational speed is lower or higher than the critical speeds of the shaft. In practice, the in-balance loads are minimized by balancing the shaft and attached components as a system. Rotordynamics theory is required if the magnitudes of the stresses at a particular operating speed is required.

### Synchronous Whirl (Due to Unbalanced Mass)

m=unbalanced mass



$$x_s = \frac{me\omega^2 \cos(\omega t - \phi)}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$y_s = \frac{me\omega^2 \sin(\omega t - \phi)}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

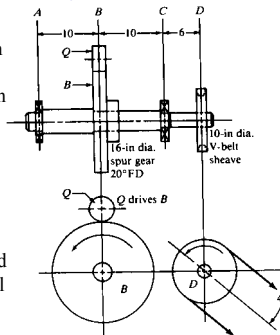
$$OS = \sqrt{x_s^2 + y_s^2} = \frac{me\omega^2}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$\tan \phi = \frac{c\omega}{k - m\omega^2}$$

Thomson, Fig. 3.4-2

### Assignment (Problem 1)

The shaft rotating at 550 rpm carries a spur gear B having 96 teeth and a diametral pitch of 6. The teeth are of the 20° full-depth, involute form. The gear receives 30 hp from a pinion directly above it.

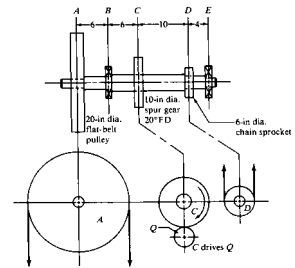


Compute the torque delivered to the shaft and the tangential and radial forces exerted on the shaft by the gear.

Mott, Fig. 12-20

### Assignment (Problem 2)

The shaft rotating at 200 rpm carries a 20-in-diameter flat-belt pulley at A that receives 10 hp from below.

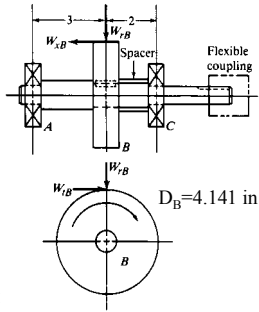


Compute the torque delivered by the pulley to the shaft and the force exerted on the shaft by the pulley.

Mott, Fig. 12-21

### Assignment (Problem 3)

The shaft is rotating at 650 rpm and receives 7.5 hp through a flexible coupling. The power is delivered to an adjacent shaft through a single helical gear B having a normal pressure angle of  $20^\circ$  and a helix angle of  $15^\circ$ .



- (a) draw free-body diagrams for the shaft in both the vertical and horizontal planes, (b) find the magnitude of the forces shown, (c) draw the shearing force and bending moment diagrams for the shaft in both planes.

Mott, Fig. 12-29

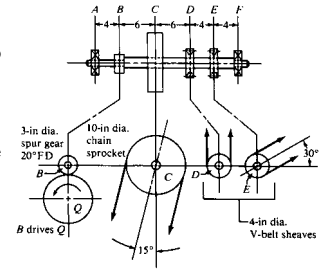


### Assignment (Problem 4)



The shaft rotating at 480 rpm carries a 10-in-diameter chain sprocket at C that receives 11 hp from a mating sprocket below and to the left as shown.

- Compute the torque delivered to the shaft by the sprocket and the total force exerted on the shaft by the sprocket. Resolve the force into its horizontal and vertical components, and show the net forces acting on the shaft at C in the vertical and horizontal directions.



Mott, Fig. 12-22