The University of Tennessee at Martin
MARTIN School of Engineering

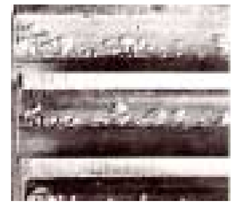
Involute Gear Tooth Contact Stress Analysis

Class 20

Engineering 473
Machine Design

Pitting In Gear Teeth

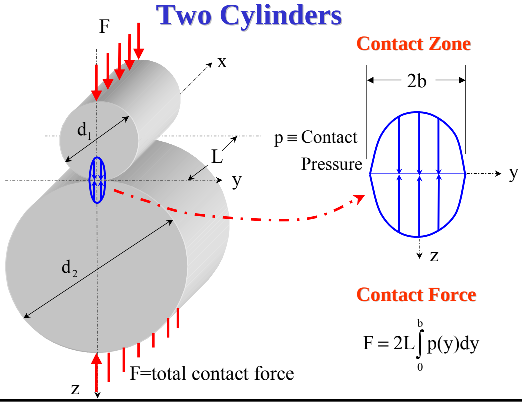
- **Pitting** – phenomenon in which small particles are removed from the surface of the tooth because of the high contact forces that are present between mating teeth.
- Pitting is actually the fatigue failure of the tooth surface.
- Hardness is the primary property of the gear tooth that provides resistance to pitting.



www.xteknic.com/products/gearproducts

Contact Stress Between Two Cylinders

Contact Zone

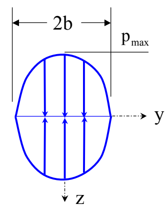


Contact Force

$$F = 2L\int_0^b p(y)dy$$

Hertz Contact Stress Equations

Contact Zone



Contact Width

$$b = \sqrt{\frac{2F}{\pi L}\,\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

Maximum Contact Pressure

$$p_{max} = \frac{2F}{\pi b L}$$

Reference Shigley, Page 73-74

Hertz Contact Stress Equations
(Continued)

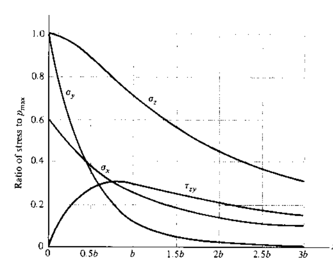
$$\sigma_x = -2\nu \cdot p_{max}\left(\sqrt{1+\frac{z^2}{b^2}} - \frac{z}{b}\right)$$

$$\sigma_y = -p_{max}\left[\left(2 - \frac{1}{1+\frac{z^2}{b^2}}\right)\sqrt{1+\frac{z^2}{b^2}} - 2\frac{z}{b}\right]$$

$$\sigma_z = \frac{-p_{max}}{\sqrt{1+\frac{z^2}{b^2}}}$$

Z-axis Stress Components

Hertz Contact Stress Equations
(Continued)

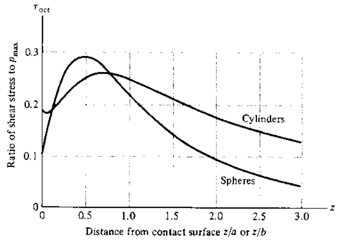


This graph shows the variation of the stress components along the z-axis.

Note that the maximum shear stress is much less than the maximum contact pressure.

Shigley, Fig. 2-35

Hertz Contact Stress Equations
(Continued)



Von Mises stress variation along the z-axis.

Note that the von Mises stress is much less than the maximum contact pressure.

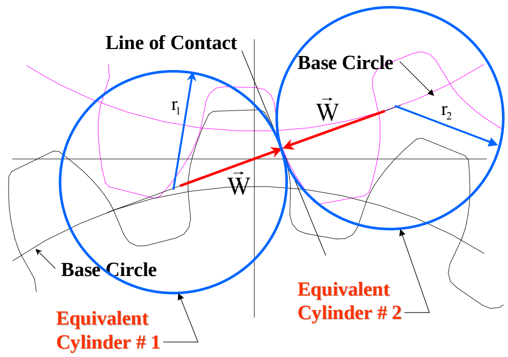
$$\sigma_{eff} = 0.26 \cdot p_{max}$$

$$\frac{S_{yt}}{N_{fs}} = 0.26 \cdot p_{max}$$

if $N_{fs} = 1.0$

then allowable $p_{max} \approx 3.85 \cdot S_{yt}$

Equivalent Contacting Cylinders



Radii of Equivalent Cylinders

$$r_1 = \frac{d_p \sin\varphi}{2}$$

$$r_2 = \frac{d_g \sin\varphi}{2}$$

$d_p \equiv$ pinion pitch diameter

$d_g \equiv$ gear pitch diameter

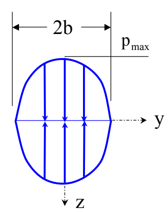
$\varphi \equiv$ pressure angle

$r_1 \equiv$ radius of equivalent cylinder #1

$r_2 \equiv$ radius of equivalent cylinder #1

Contact Stress in Gear Teeth

Contact Zone



Contact Width

$$b = \sqrt{\frac{2F}{\pi L} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

Maximum Contact Pressure

$$p_{max} = \frac{2F}{\pi b L}$$

Contact Stress

$$p_{max} = -\frac{2F}{\pi b L}$$

$$b = \sqrt{\frac{2F}{\pi L} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

$$b = \sqrt{\frac{F}{L}} \sqrt{\frac{1}{\pi}\left[\frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2}\right]} \sqrt{\frac{4}{\frac{1}{r_1} + \frac{1}{r_2}}}$$

Elastic Coefficient

$$C_p = \left[\frac{1}{\pi\left(\frac{1-\nu_p^2}{E_p} + \frac{1-\nu_g^2}{E_g}\right)}\right]^{\frac{1}{2}}$$

$$p_{max} = -C_p\left[\frac{F}{L}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right]^{1/2}$$

Contact Stress
(Continued)

$$p_{max} = -C_p\left[\frac{F}{L}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right]^{1/2}$$

$$F = W_n = W_t / \cos\varphi$$

$$\sigma_c = -C_p\left[\frac{W_t}{L\cos(\varphi)}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right]^{1/2}$$

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{\sin\varphi}\left(\frac{1}{d_p} + \frac{1}{d_g}\right)$$

$$m_g \equiv \text{speed ratio} = \frac{d_g}{d_p} \text{ (external gears)}$$

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{d_p \sin\varphi}\left(\frac{m_g + 1}{m_g}\right)$$

$$\sigma_c = -C_p\left[\frac{W_t}{d_p L I}\right]^{1/2}$$

$$I = \frac{\cos\varphi \cdot \sin\varphi}{2} \frac{m_g + 1}{m_g}$$

Contact Stress Summary

$$\sigma_c = -C_p\left[\frac{W_t}{d_p L I}\right]^{1/2}$$

Form Factor

$$I = \frac{\cos\varphi \cdot \sin\varphi}{2}\,\frac{m_g + 1}{m_g}$$

Elastic Coefficient

$$C_p = \left[\frac{1}{\pi\left(\frac{1-\nu_p^2}{E_p} + \frac{1-\nu_g^2}{E_g}\right)}\right]^{\frac{1}{2}}$$

$m_g = \frac{d_g}{d_p}$ (external gears)

$d_g \equiv$ gear pitch diameter

$d_p \equiv$ pinion pitch diameter

$\varphi \equiv$ pressure angle

$L \equiv$ face width

$W_t \equiv$ tangential force

AGMA Contact Stress Formulas

$$\sigma_c = C_p\left(\frac{W_t C_a}{C_v}\,\frac{C_s}{Fd}\,\frac{C_m C_f}{I}\right)^{1/2}$$

$$\sigma_{c,all} = \frac{S_c C_L C_H}{C_T C_R}$$

$\sigma_c \equiv$ absolute value of contact stress

$C_p \equiv$ elastic coefficient

$C_a \equiv$ application factor

$C_v \equiv$ dynamic factor

$C_s \equiv$ size factor

$d \equiv$ pitch diameter of pinion

$C_m \equiv$ load – distribution factor

$C_f \equiv$ surface - condition factor

$I \equiv$ geometry factor

$\sigma_{c,all} \equiv$ allowable contact stress

$C_L \equiv$ life factor

$C_H \equiv$ hardness ratio factor

$C_T \equiv$ temperature factor

$C_R \equiv$ reliability factor

Allowable Contact Stress Values

TABLE 9–3 Allowable stress numbers for case-hardened steel gear materials

	Allowable bending stress number, s_{at} (Ksi)			Allowable contact stress number, s_{ac} (Ksi)		
Hardness at surface	Grade 1	Grade 2	Grade 3	Grade 1	Grade 2	Grade 3
Flame- or induction-hardened:						
50 HRC	45	55		170	190	
54 HRC	45	55		175	195	
Carburized and case-hardened:						
55–64 HRC	55			180		
58–64 HRC	55	65	75	180	225	275
Nitrided, through-hardened steels:						
83.5 HR15N		See Figure 9–12.		150	163	175
84.5 HR15N		See Figure 9–12.		155	168	180
Nitrided, nitralloy 135M:[a]						
87.5 HR15N		See Figure 9–13.				
90.0 HR15N		See Figure 9–13.		170	183	195
Nitrided, nitralloy N:[a]						
87.5 HR15N		See Figure 9–13.				
90.0 HR15N		See Figure 9–13.		172	188	205
Nitrided, 2.5% chrome (no aluminum):						
87.5 HR15N		See Figure 9–13.		155	172	189
90.0 HR15N		See Figure 9 13.		176	196	216

SOURCE: Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

[a]Nitralloy is a proprietary family of steels containing approximately 1.0% aluminum which enhances the formation of hard nitrides.

Hardness Ratio Factor

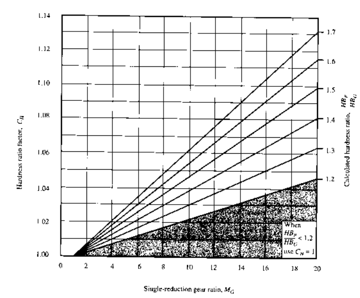


FIGURE 9–23 Hardness ratio factor, C_H (through-hardened) (Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

Assignment

Web reading assignments

www.xtekinc.com/products/gearproducts

(look under product information)

http://hghouston.com/case012.html

1. A speed reducer has 20 degree full-depth teeth and consists of a 22-tooth steel spur pinion driving a 60-tooth cast-iron gear. The horsepower transmitted is 15 at a pinion speed of 1200 rev/min. For a diametral pitch of 6 teeth/in and a face width of 2 in, find the contact stress.
2. A gearset has a diametral pitch of 5 teeth/in, a 20 degree pressure angle, and a 24-tooth cast iron spur pinion driving a 48-tooth cast iron gear. The pinion is to rotate at 50 rev/min. What horsepower input can be used with this gearset if the contact stress is limited to 100 kpsi? and F=2.5 in?