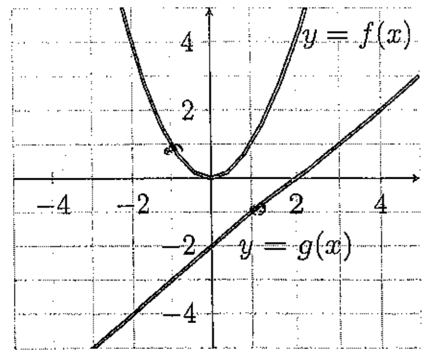
Math 140 Test Three ____________________

This fun test covers Chapter Six sections 1 through 8 of Algebra and Trigonometry (2^{nd} UTM edition) by Sullivan and Sullivan. Clearly $\boxed{\text{indicate}}$ your answers—no credit will be given for answers that I cannot find or cannot read. Unless otherwise indicated, all parts of problems are four points each.

1. The graphs of $y = f(x)$ and $y = g(x)$ are shown on the right. Evaluate $g\circ f(-1)$. (2 points)



$$g(f(-1)) = g(1) = \boxed{-1}$$

2. For the functions $f(x) = 3x + 2$ and $g(x) = x^2 - 3$ find $g\circ f(4)$.

$$g(f(4)) = g(3\cdot 4+2) = g(14) = 196-3 = \boxed{193}$$

3. For the functions $f(x) = \dfrac{3}{x-1}$ and $g(x) = \dfrac{2}{x}$ find $f\circ f(x)$.

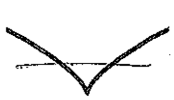
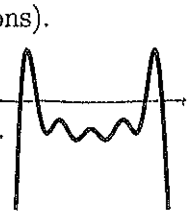
$$f(f(x)) = f\left(\frac{3}{x-1}\right) = \frac{3}{\frac{3}{x-1} - 1}\cdot\frac{x-1}{x-1} = \frac{3(x-1)}{3-(x-1)}$$

$$= \boxed{\frac{3x-3}{4-x}}$$

4. Determine whether the function is one-to-one. $\{(2,6), (-3,6), (4,9), (1,10)\}$. (2 points)

$\boxed{\text{No}}$ $(f(2) = f(-3) = 6)$

5. Use the horizontal line test to determine which of the following functions are one-to-one (circle the letters of the one-to-one functions).

A.  B.  C.  D. 

6. The function $f(x) = x^2 + 2$ ($x \geq 0$) is one-to-one. Find the inverse function f^{-1}.

$y = x^2 + 2$

exchange: $x = y^2 + 2 \quad (y \geq 0)$

$y^2 = x - 2$

$y = \boxed{\sqrt{x-2} = f^{-1}(x)}$

7. The function $f(x) = \dfrac{7}{x+3}$ is one-to-one.

(a) Find the domain of f. (2 points)

$\boxed{x \neq -3}$ or $\boxed{(-\infty, -3) \cup (-3, \infty)}$

(b) Find the inverse function f^{-1}.

$y = \dfrac{7}{x+3} \Rightarrow x = \dfrac{7}{y+3}$ so $y + 3 = \dfrac{7}{x}$

and $y = \boxed{\dfrac{7}{x} - 3 = f^{-1}(x)}$

(c) Find the domain of f^{-1}. (2 points)

$\boxed{x \neq 0}$ or $\boxed{(-\infty, 0) \cup (0, \infty)}$

8. Solve the equation: $2^{2x} = 2^6$

same base so $2x = 6$

$\boxed{x = 3}$

9. Solve the equation $5^{x+2} = 25$.

$5^{x+2} = 5^2$ so $x + 2 = 2$ and $\boxed{x = 0}$

10. Evaluate the logarithm $\log_4 \frac{1}{\sqrt{16}}$ exactly without using a calculator. (2 points)

$\log_4 \frac{1}{4} = \log_4 4^{-1} = \boxed{-1}$

11. Write $(1/4)^{-1/2} = 2$ in logarithmic form.

$$\boxed{\log_{1/4} 2 = -\frac{1}{2}}$$

12. Approximate $\log_7 9$.

$$\frac{\ln 9}{\ln 7} \approx \boxed{1.1292}$$

13. Solve the equation $\log_x 4 = 2$.

switch forms: $x^2 = 4$ so $\boxed{x = 2}$

14. Write $\ln x^3 = 10$ in exponential form.

$$\boxed{x^3 = e^{10}}$$

15. Find the domain of $f(x) = 6\log_4(2-x)$.

z

$2 - x > 0$ so $\boxed{x < 2}$ or $(-\infty, 2)$

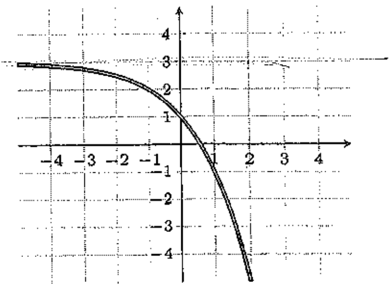
16. Find the asymptote of $f(x) = \log_4(2-x)$. (Be sure to write "$y =$" or "$x =$" as appropriate.)

$\boxed{X = 2}$

17. The graph on the right is the graph of which of the following functions? (Circle the answer.)

A. $-2^{x-3}+3$ B. $\cancel{2^{x+3}-3}$ C. $\cancel{2^{x}-3}$ $\boxed{\text{D. } -2^{x+1}+3}$

(2 points)



18. Write $\ln(x^2y^4)$ as a sum or difference of logarithms. Write the powers as factors.

$$\ln x^2 + \ln y^4 = \boxed{2\ln x + 4\ln y}$$

19. Solve the logarithmic equation: $\ln(-x) + \ln(1-x) = \ln(-2x)$.

$$\ln(-x(1-x)) = \ln(-2x)$$

so

$$-X + X^2 = -2X$$

which is

$$X^2 + X = X(X+1) = 0$$

so $\cancel{X = 0}$ or $\boxed{X = -1}$

20. Solve the exponential equation: $2 \cdot 3^x + 1 = 10$.

$$2 \cdot 3^x = 9$$

$$3^x = 9/2$$

$$\boxed{X = \log_3 9/2 = \frac{\ln 9/2}{\ln 3} \approx 1.369}$$

21. If \$1000 is placed in a bank at 4% compounded continuously, how much will there be after 5 years?

$$A = Pe^{rt}$$

$$= 1000\, e^{(.04)5} = \boxed{\$1221.40}$$

22. How long will it take \$1000 dollars to grow to \$1127 when the interest rate is 3% compounded monthly?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$1127 = 1000\left(1 + \frac{.03}{12}\right)^{12t}$$

$$\ln 1.127 = 12t \ln\left(1 + \frac{.03}{12}\right)$$

so $t \approx \boxed{3.99 \text{ years}}$

23. How long must \$1000 be in a bank at 2% compounded annually to become \$1319.50 (round to the nearest tenth of a year).

(why same type of problem?)

$$1319.50 = 1000\left(1 + \frac{.02}{1}\right)^{1 \cdot t}$$

$$\ln 1.3195 = t \ln(1.02)$$

$$\boxed{t = 14}$$

24. The radioactive material Mathium decays exponential according to the function

$$f(t) = 100e^{-0.04t}$$

where t is the time in days and $f(t)$ is the quantity in kilograms. Approximately how long will it take for the quantity of Mathium to decay to 70 kilograms?

-1 for 9 with no other work

$$70 = 100e^{-0.04t}$$

$$.70 = e^{-0.04t}$$

$$\ln(.70) = -.04t$$

$$t = -\frac{\ln(0.70)}{0.04} \approx \boxed{8.92 \text{ days}}$$

25. The table list the percentage of the U.S. population which are zombies. Here x is the number of years since 2000 (so x=16 is 2016) and y be the percentage. Use the exponential regression feature of your calculator to determine the exponential function of best fit.

year	percent
0	1.0
5	2.3
10	4.5
15	12.6

a) Write the equation.

$$y = 0.977(1.18)^x$$

b) What percentage of the population will be zombies in 2020 ($x = 20$)? (2 points)

$$y(20) = 0.977(1.18)^{20} = 26.8\%$$