Math 241

Test Four

This enjoyable fifty-minute 25 point test, covers induction. All proofs are five points unless otherwise stated. Prove five of the following seven statements. Place each proof on a separate page.

a. $1\cdot 1! + 2\cdot 2! + 3\cdot 3! + \ldots + n\cdot n! = (n+1)! - 1$

b. 3 is a divisor of $2^{2n} - 1$.

c. If $a \in \Re \setminus \{0\}$, $\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}^n = \begin{pmatrix} a^n & na^{n-1} \\ 0 & a^n \end{pmatrix}$

d. If $n \geq 5$, then $n^2 < 2^n$.

e. If $x > -1$, then $(1 + x)^n \geq 1 + nx$.

f. $1 + x + x^2 + x^3 + \ldots + x^n = \dfrac{1 - x^{n+1}}{1 - x}$ $(x \neq 1)$.

g. Every positive integer can be written as a product of primes. [By convention, we say one is a product of zero primes—so an empty product equals one. Similarly an empty sum equals zero.]