

Math 241

This fun fifty-minute test covers the first two chapters of “Chapter Zero” by Carol Schumacher. Note that the universe for the variables x , y and z is the set of reals, and for n is the set of positive integers. All parts of problems are five points unless otherwise stated.

1. Euclid called his five assumptions for geometry “common notions.” We now call them axioms or postulates. State briefly why he called them common notions and we do not.
2. For each of the following, indicate that it is a “predicate,” “statement” or “neither.” In one sentence (each) state why.
 - a. $\forall x > 0, x^2 = 1$.
 - b. $x^n + y^n = z^n$.
 - c. Mr. Eskew is a very good teacher.
3. Find the contrapositive of the following implication: “If man is saved by grace and man is not saved by works, then the neopelagianists are wrong.”

4. Find the negation of the following implication: If $y > 0$, then $\forall x \exists z$ such that $x^y = z$.

5. List the six ways to bind the variables in the following predicate using the universal quantifiers \forall, \exists to make it a statement: $xy^2 \leq 1$. Next to each state its truth value. (3 points each)

a.

b.

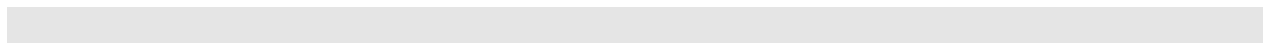
c.

d.

e.

f.

6. Show the following is a tautology: $\sim (A \Rightarrow B) \Leftrightarrow (A \wedge \sim B)$. (10 points)



Definition 1: If A and B are sets, we say that A is a **subset** of B (denotes $A \subseteq B$) iff every element of A is an Element of B . (So $A \subseteq B$ is equivalent to “if $x \in A$, then $x \in B$.”)

Definition 2: A set with no elements is called an **empty set** (denoted \emptyset or $\{ \}$.)

Use the definitions above to prove the following. Use either contraposition or contradiction on the third. (10 points each.)

Theorem One: For all sets A , $\emptyset \subseteq A$.

Theorem Two: For all sets A , $A \subseteq A$.

Theorem Three: If $A \subseteq \emptyset$, then A is an empty set.