Math 241

This fun fifty-minute test covers chapters four and five of "Chapter Zero" by Carol Schumacher. All parts of problems are five points unless otherwise stated.

1. Circle T for (always) true or F for (at least once) false (two points each)
   T F A function from A to A is a relation on A.

   T F A relation on the set A is a function from A to A.

   T F A binary operation on A is a function from A to A.

   T F A total ordering on A is a binary operation on A.

   T F If the functions $f:A\rightarrow B$ and $g:B\rightarrow A$ have inverses, then $f \circ g$ is defined.

   T F If the functions $f:A\rightarrow B$ and $g:A\rightarrow B$ have inverses, then fg has an inverse.

   T F If $f:A\rightarrow A$ and the range of f equals the domain of f, then f is surjective.

   T F If $f:A\rightarrow A$ and the range of f equals the domain of f, then f is injective.
2. Give an example of a relation on the set A = {1,2,3} which is an equivalence relation, a partial ordering and a function.
3. Prove that a total ordering on A = {1,2,3} is not a function.
4. Define a binary operation * on the reals by x*y = z where

   $$(1+x^2)\,(1+y^2)\,(1+z^3) = 1.$$

   Either prove or disprove the following:

   a. Theorem: * is associative.
   b. Theorem: * is commutative.
5. Define a relation ~ on the set of reals R by a~b iff 3a+5b is even (is twice an integer).
   a. Prove or disprove that ~ is an equivalence relation on R.
   b. Prove or disprove that ~ is an equivalence relation on the set of positive integers N.