Math 241

Test Two ______________________

This fun fifty-minute test covers sections 1, 2, and 3 of our set theory supplement. All parts of problems are six points unless otherwise stated.

1. Circle T for (always) true or F for (at least once) false (two points each)

   T F We must begin an axiomatic system with undefined terms or relations.

   T F To prove $p \rightarrow q$ by contradiction, we first assume $q \rightarrow p$.

   T F To prove $p \rightarrow q$ by contraposition, we first assume $\sim q \rightarrow \sim p$.

   T F With our present axioms we have the ability to prove at least two sets exist.

   T F The set {1,2,3,4,5,6,7,8} has more than 400 subset..

   T F This is my favorite class.

   T F If A={1,2,3,4,5,6,7,8} and B={1,2,3,4}, then A\B=B\∅.

Prove the following:

2. **Theorem 2.2**.

3. **Theorem 2.4ii**.

4. **Theorem 3.3 iv**.

So far our axioms guarantee the existence at least one set, and selected subsets of existing sets. This is not enough to be able to define sets which are supersets, such as $A \cup B$ (a superset of A and of B). The next axiom is our first that allows us to join sets into new sets.

Axiom IV (Pairing Axiom): If A and B are sets, then there is a set C such that $A \in C$ and $B \in C$.

Theorem 4.1: Let A and B be sets. There exists a unique set (denoted $\{A,B\}$) which contains only A and B as elements.

(Hint: The axiom says A and B are in C, but do not say they are the only things in C, so you can start with C and specify the right subset…)

5. Prove one of the following two theorems:

   Theorem 4.2: If A and B are sets, then {A, B} = {B, A}.

   Proposition 4.3: Let A be a set. The set {A} (a set whose only element is A) is well defined.