Math 241

A Wonderful, 2 hour, Mathematics 241 final for 1997.

1. Give an example of a predicate.

2. Here the definition of limit from calculus:

   $$\forall \varepsilon>0\ \exists \delta>0 \text{ such that } 0 < | x - x_0 | < \delta \text{ implies } | f(x) - L | < \varepsilon.$$

   State the negation of this expression.

3. Let A be the statement "If I am Bond, then I am James Bond."

   a. State the converse of A.

   b. State the contrapositive of A.

4. Construct a truth table for the following expression and determine if it is a tautology.

   $$(A \Rightarrow B) \Leftrightarrow (B \vee \sim A)$$

5. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Find $f^{-1}([0,1])$.

6. Find the gcd of 234 and 208 using the Euclidean algorithm.

7. Prove that if $A \subseteq B$ and $B \subseteq A$, then $A = B$.

8. Define a relation on $\mathbb{Z}$ (the integers) by "$a \sim b$ iff seven divides $2a+5b$."
   Prove or disprove that this is an equivalence relation.

9. Two sets A and B are "equivalent" iff there is a bijection f from A to B.

   a. Show (by finding a bijection) that the set of integers is equivalent to the subset of even integers.

   b. Prove this is an equivalence relation on the universe of all sets.

10. Prove that for all positive integers n, $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + n \cdot n! = (n+1)! - 1$.

11. Any integer postage greater than 11 cents can be formed using 4-cent and 5-cent stamps.