1. The value of $\lim_{x \to 5} \left\{ \frac{3x^2 - 13x - 10}{x^2 - 25} \right\}$ is:

   a) 0

   b) 1

   c) 3

   d) 1.7

   e) not defined

2. The value of $\lim_{x \to -\infty} \left(\frac{5 \sin x}{x} \right)$ is:

   a) 1

   b) 5

   c) undefined

   d) $-\infty$

   e) 0

3. The function y = f(x) has a nonremovable (jump) discontinuity at x = a if

   a) $\lim_{x \to a} f(x) = 3$ and f(a) = 5

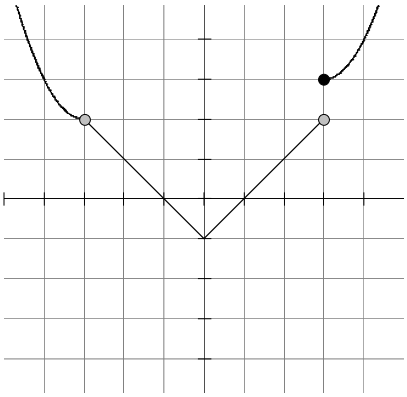
   b) $\lim_{x \to a^-} f(x) = 3$ and $\lim_{x \to a^+} f(x) = 5$

   c) $\lim_{x \to a^-} f(x) = \infty$ and $\lim_{x \to a^+} f(x) = -\infty$

   d) $\lim_{x \to a^-} f(x) = 3$ and $\lim_{x \to a^+} f(x) = \infty$

   e) $\lim_{x \to a} f(x) = 3$ and f(a) = 3

Problems 4 and 5 use the following graph of a function:



4. The value of $\lim_{x \to 3^+} f(x)$ is:

   a) 3 b) 1 c) 2 d) -1 e) undefined

5. To remove the discontinuity at x = -3, define a new function g(x) by:

$$g(x)=\begin{cases} f(x) & \text{if } x \neq -3 \\ k & \text{if } x = -3 \end{cases}.$$

   What is the value of k?

   a) -1 b) 2 c) 0 d) -3 e) The discontinuity cannot be removed.

6. Find the first derivative of $y = \sec (10x^3 - 11x - 5)$.

   a) $(30x^2 - 11) \sec x \tan x$

   b) $\sec (10x^3 - 11x - 5) \tan (10x^3 - 11x - 5)$

   c) $\sec (30x^2 - 11) \tan (30x^2 - 11)$

   d) $(30x^2 - 11) \sec (10x^3 - 11x - 5) \tan (10x^3 - 11x - 5)$

   e) $-(30x^2 - 11) \sec (10x^3 - 11x - 5) \tan (10x^3 - 11x - 5)$

7. Find $\frac{d^{25}y}{dx^{25}}$ if y = cos x.

a) -sin x

b) -cos x

c) sin x

d) cos x

e) 0

8. The equation of the line tangent to the graph of $f(x) = \sqrt{15 - x}$ at the point (11, 2) is

a) $y = -\frac{1}{4}x + \frac{19}{4}$

b) $y = -\frac{1}{4}x + \frac{11}{4}$

c) $y = -\frac{1}{4}$

d) $y = \frac{1}{4}x + \frac{11}{4}$

e) $y = \frac{1}{4}x - \frac{3}{4}$

9. The derivative of $f(x) = (2x^3 + x + 5)^4$ is:

a) $4\,(6x^3 + 1)^3$

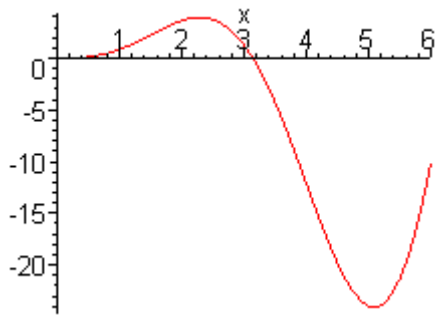
b) $\frac{1}{5}\,(6x^3 + 1)^5$

c) $\frac{1}{5}\,(2x^3 + x + 5)^5$

d) $4\,(2x^3 + x + 5)^3\,(6x^2 + 1 + 5)$

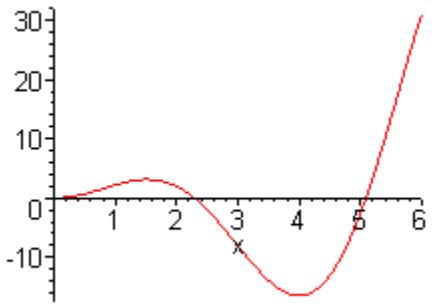
e) $4\,(2x^3 + x + 5)^3\,(6x^2 + 1)$

10. The graph of the first derivative of a function is shown below:

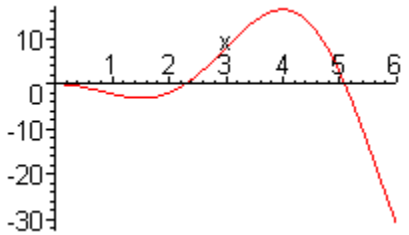


Which of the following graphs could be the graph of the original function?

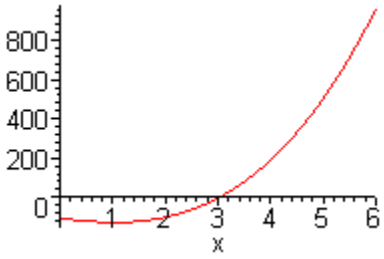
a)



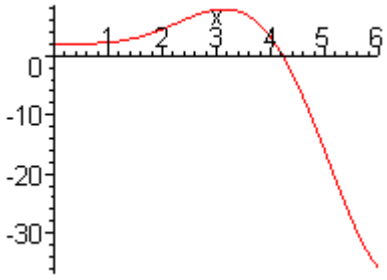
b)



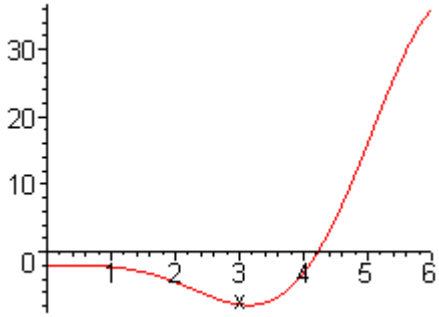
c)



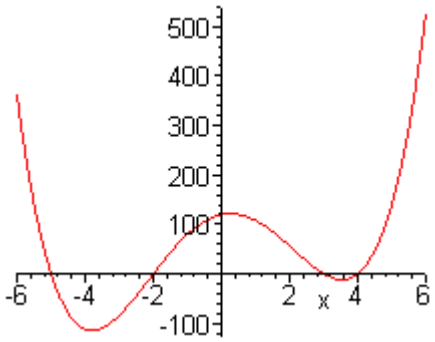
d)



e)

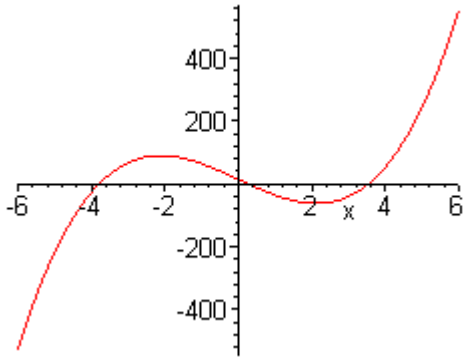


11. The graph of a function is shown below:

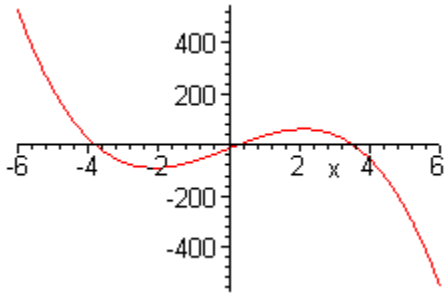


Which of the following graphs could be the graph of the derivative of the function?

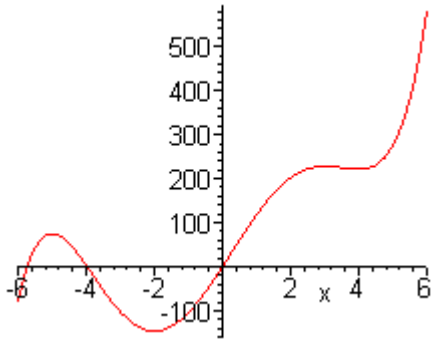
a)



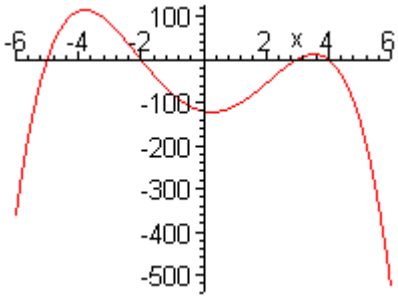
b)



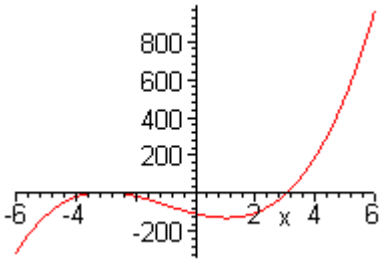
c)



d)



e)



12. Find $\frac{dy}{dx}$ if $y^2 + xy + x^2 = 10$.

a) $\frac{dy}{dx} = \frac{-3x}{2y}$

b) $\frac{dy}{dx} = \frac{-2x-2y}{x}$

c) $\frac{dy}{dx} = \frac{-2x-y}{2y+x}$

d) $\frac{dy}{dx} = \frac{-2x}{2y+x}$

e) $\frac{dy}{dx} = \frac{2x+y}{2y+x}$

13. The absolute maximum value of $f(x) = (x + 2)^2 (19 - x)$ over the interval $[-6, 0]$ is:

a) 72

b) 76

c) 400

d) 1372

e) 40572

14. Let $f(x) = \frac{x}{x^2+1}$. The number of inflection points of the graph of f(x) is:

a) none

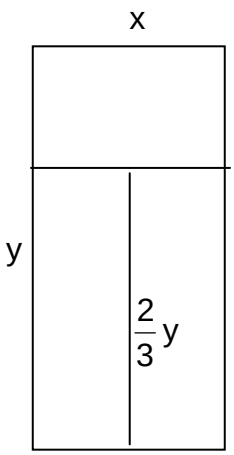
b) one

c) two

d) three

e) four

15. A rancher has 1600 feet of fencing in which to enclose three corrals as shown in the diagram. What dimensions (rounded to the nearest hundredth) should the corrals have in order to enclose the maximum area?



a) x = 266.67 feet
   y = 266.67 feet

b) x = 266.67 feet
   y = 300 feet

c) x = 300 feet
   y = 262.5 feet

d) x = 262.50 feet
   y = 300 feet

e) x = 200 feet
   y = 375 feet

16. A hot air balloon is floating at a constant altitude of 400 feet when it passes over a small child. The wind is blowing east at a rate of 15 feet per second. How fast (rounded to two decimal places) is the distance between the child and the balloon increasing after 10 seconds?

a) 5.27 feet per second

b) 26.67 feet per second

c) 15.00 feet per second

d) 427.20 feet per second

e) 28.48 feet per second

17. Estimate the area under the curve $y = x^3 + x$ on the interval [1, 3] with n = 4 subintervals using the midpoint of each subinterval.

a) 35 b) 47.5 c) 65 d) 80.5 e) 23.75

18. Find $\frac{d}{dx}\int_2^x \frac{dt}{\sqrt{t^2-1}}$.

a) $\frac{1}{\sqrt{x^2-1}}$

b) $\frac{1}{\sqrt{x^2-1}}-\sqrt{3}$

c) $\frac{2x}{\sqrt{x^2-1}}$

d) $\sqrt{x^2-1}-\sqrt{3}$

e) $\frac{x}{\sqrt{x^2-1}}$

19. Evaluate $\int \frac{x^4+1}{x^2}dx$.

a) $\frac{1}{3}x^3-\frac{1}{3}x^{-3}+C$

b) $\frac{1}{3}x^3-\frac{1}{x}+C$

c) $\frac{1}{3}x^3+\frac{1}{x}+C$

d) $\frac{\frac{1}{5}x^5+x}{\frac{1}{3}x^3}+C$

e) $\frac{1}{3}x^3+x+C$

20. Find $\int_1^a 12x^2 - 6x + 4\,dx$.

a) $12a^3 - 6a^2 + 4a - 10$

b) $4a^3 - 3a^2 + 4a - 5$

c) $12a^3 - 6a^2 - 6$

d) $4a^3 - 3a^2 - 1$

e) $4a^3 - 3a^2 + 4a + 5$

21. Find $\int x\,(x^2 + 3)^{16}\,dx$.

a) $\frac{1}{2}x^2\left(\frac{x^3}{3} + 3x\right)^{16} + C$

b) $\frac{1}{34}x^2(x^2 + 3)^{17} + C$

c) $\frac{x^{49}}{49} + \frac{3x^{17}}{17} + C$

d) $\frac{1}{34}(x^2 + 3)^{17} + C$

e) $\frac{1}{17}(x^3 + 3x)^{17} + C$

22. Find $\int_1^3 \frac{x}{\sqrt{1+3x^2}}dx$.

a) $\sqrt{7}-1$

b) $\frac{2}{3}\left(\sqrt{7}-1\right)$

c) 0

d) $\sqrt{7}$

e) Does not exist

23. The velocity of an object at any time t (in seconds) is given by: $v(t) = t^2 + \cos t$. Initially, the object is at position s = 10 feet. What is the object's position (rounded to two decimal places) 8 seconds later?

a) 171.66

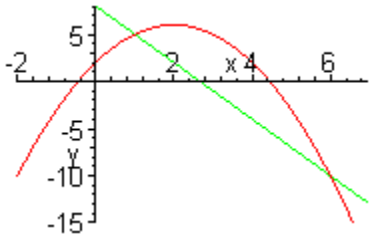
b) 169.68

c) 175.67

d) 179.68

e) 181.66

Problems 24-26 refer to the figure which is the graph of $y = -x^2 + 4x + 2$ and $y = -3x + 8$.



24. The area of the region bounded by $y = -x^2 + 4x + 2$ and $y = -3x + 8$ is

a) $5\frac{1}{6}$ square units

b) $9\frac{5}{6}$ square units

c) $20\frac{5}{6}$ square units

d) $84\frac{1}{6}$ square units

e) $224\frac{1}{6}$ square units

25. A correct integral for finding the volume of the solid obtained by rotating the region bounded by $y = -x^2 + 4x + 2$ and $y = -3x + 8$ about the y-axis using the cylindrical shell method is :

a) $\int_1^6 2\pi x\,(-x^2 + x - 6)\,dx$

b) $\int_1^6 2\pi x\,(-x^2 + 7x - 6)\,dx$

c) $\int_1^6 \pi\,(-x^2 + x - 6)^2\,dx$

d) $\int_1^6 \pi\,(-x^2 + 7x - 6)\,dx$

e) $\int_1^6 [\pi\,(-x^2 + 4x + 2) - \pi\,(-3x + 8)^2]\,dx$

26. A correct integral for finding the volume of the solid obtained by rotating the region bounded by $y = -x^2 + 4x + 2$ and $y = -3x + 8$ about the line y = -13 is :

a) $\int_1^6 \pi(-x^2 + x - 19)^2 \, dx$

b) $\int_1^6 \pi(-x^2 + 7x + 7)^2 \, dx$

c) $\int_1^6 \pi(-x^2 + 7x - 6)^2 \, dx$

d) $\int_1^6 [\pi(-x^2 + 4x + 15)^2 - \pi(-3x + 21)^2] \, dx$

e) $\int_1^6 [\pi(-x^2 + 4x - 11)^2 - \pi(-3x - 5)^2] \, dx$

27. Find the average value of the function $f(x) = 6x^2 - 4x + 1$ on the interval [0,5].

a) $\frac{131}{2}$ b) 75 c) 5 d) 205 e) 41

28. A rectangular tank with base 3 feet by 8 feet and height 12 feet is full of water. Find the work required to pump the water to a point 5 feet above the top of the tank. Assume the density of water is 62.5 lb/ft^3 .

a) 8250 ft-lbs

b) 198000 ft-lbs

c) 108000 ft-lbs

d) 89250 ft-lbs

e) 216750 ft-lbs

29. Find the derivative of f(x) = sinh x tanh x.

a) f ' (x) = sinh x sech^2 x + tanh x $\cosh^2$ x

b) f ' (x) = sinh x sech^2 x + tanh x

c) f ' (x) = sinh x sech^2 x + tanh x cosh x

d) f ' (x) = sinh x sech x + tanh x cosh x

e) f ' (x) = sinh x sech x + tanh x $\cosh^2$ x

30. Define f(x) = $\int_1^x \frac{dt}{t}$ for x > 0. Which of the following is not true

a) $\frac{df(x)}{dx} = \frac{1}{x}$

b) f(x) is an increasing function

c) f(x) is a one to one function

d) f(1) = 0

e) f(x) $\geq$ 0 for all x

31. Find the area of the surface obtained by rotating the curve y = 2x over the interval [0,1] about the y-axis.

a) $\frac{6\pi}{5}$

b) $\frac{\pi}{5}$

c) $2\pi\sqrt{5}$

d) $\pi\sqrt{5}$

e) $\frac{\pi}{\sqrt{5}}$

No longer in Math 251,

this is now in Math 252

So not on your final.

32. Find the exact values of the numbers c that satisfy the conclusion of The Mean Value Theorem for the function $f(x) = x^3 - 2x$ for the interval [-2, 2].

a) $\pm 2\sqrt{3}$ b) $\pm \frac{2\sqrt{3}}{3}$ c) $\frac{2\sqrt{3}}{3}$ d) $-\frac{2\sqrt{3}}{3}$ e) $2\sqrt{3}$

33. Find the absolute minimum values of $y = 10x^2 - 40x + 8$ on the interval [0, 3].

a) -32 b) 2 c) 32 d) 40 e) –40

34. Find the intervals on which the following function f is increasing:

$f(x) = x^3 - 192x + 5$

a) $(-\infty, 24) \cup (24, \infty)$

b) (-8, 8)

c) $(-\infty, -8) \cup (8, \infty)$

d) $(-\infty, 8)$

e) $(-8, \infty)$

35. Under ideal conditions a certain bacteria population is known to double every three hours. Suppose that there are initially 100 bacteria. What is the size of the population after 15 hours?

a) 3200 bacteria

b) 3220 bacteria

c) 6400 bacteria

d) 1600 bacteria

e) 13700 bacteria

36. Find the points on the curve $y = 2x^3 + 3x^2 - 36x + 3$ where the tangent line is horizontal.

a) (-2, 71) and (2, -41)

b) (-3, 84) and (2, -41)

c) (-2, 71), (2, -41), (-3, 84), and (3, -24)

d) (-3, 84) and (3, -24)

e) (-2, 71) and (3, -24)

37. Differentiate the function: $f(t) = \ln(\cos(6t))$.

a) $f'(t) = \dfrac{6}{\cos(6t)}$

b) $f'(t) = -6 \cot(6t)$

e) $f'(t) = 6 \sec(6t)$

d) $f'(t) = \dfrac{1}{\cos(6t)}$

e) $f'(t) = -6 \tan(6t)$

38. At noon, ship A is 160 km west of ship B. Ship A is sailing south at 32 km/hr and ship B is sailing north at 30 km/h. How fast is the distance between the ships changing at 4:00 p.m.? Round the result to the nearest thousandth if necessary.

a) 52.099 km/h

b) 52.208 km/h

c) 52.048 km/h

d) 52.098 km/h

e) 52.096 km/h

39. Use Newton's Method with the initial approximation $x_1 = 10$ to find x_4, the fourth approximation to the root of the equation: $x^3 + x^2 - 6 = 0$.

a) $x_4 = 6.09191$

b) $x_4 = 2.88191$

c) $x_4 = 1.39225$

d) $x_4 = 4.32854$

e) $x_4 = 1.44663$

40. Differentiate the function: $f(t) = 3\sqrt{t} - \frac{9}{\sqrt{t}}$.

a) $f'(t) = \frac{5t\sqrt{t} - 9\sqrt{t}}{2}$

b) $f'(t) = \frac{3\sqrt{t}}{2} + \frac{11}{2t\sqrt{t}}$

c) $f'(t) = \frac{3}{\sqrt{t}} - \frac{9}{2t\sqrt{t}}$

d) $f'(t) = \frac{1}{2\sqrt{t}} - \frac{11}{2t\sqrt{t}}$

e) $f'(t) = \frac{3}{2\sqrt{t}} + \frac{9}{2t\sqrt{t}}$