

This easy fifty-minute test covers sections 2.1 through 3.2 of *Calculus* by J. Stewart. Clearly indicate your answers—no credit will be given for answers that I can find or read. Unless otherwise indicated, all parts of problems are three points each.

1. Complete the  $\delta$ - $\epsilon$  definition of limit:  $\lim_{x \rightarrow a} f(x) = L$  if for

2. State the (limit) definition of derivative.

3. Sketch (on the right) the graph of a function  $f$  that satisfies the following five conditions. (5 points)

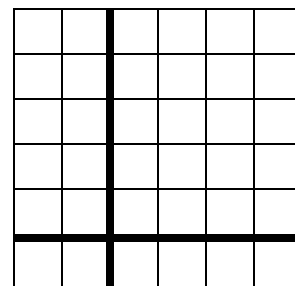
$$f(1) = 1$$

$$f(0) = 0$$

$$\lim_{x \rightarrow 0^+} f(x) = 2$$

$$\lim_{x \rightarrow 0^-} f(x) = 1$$

$$\lim_{x \rightarrow 1} f(x) = 0$$



4. Given that  $\lim_{x \rightarrow a} f(x) = 3$ ,  $\lim_{x \rightarrow a} g(x) = 2$  and  $\lim_{x \rightarrow a} h(x) = 4$  find the following limits. If the limit does not exist, explain why.

a.  $\lim_{x \rightarrow a} (3f(x) - 2g(x))$

b.  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$

c.  $\lim_{x \rightarrow a} (f(x)g(x) + 8h(x))$

d.  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)^2 - h(x)}$

5. Evaluate the following limits

a.  $\lim_{x \rightarrow 2} 2^{-x}$

b.  $\lim_{x \rightarrow 2} \sin(\pi x)$

c.  $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1}$

d.  $\lim_{x \rightarrow 1} \frac{x + 1}{x^2 - 1}$

e.  $\lim_{x \rightarrow 4^+} \frac{x-4}{|x-4|}$

6. Let  $f(x) = \begin{cases} 4x-3 & \text{if } x \geq 2 \\ x^2 & \text{if } 0 \leq x < 2. \\ 1/x & \text{if } x < 0 \end{cases}$  Find the following limits.

a.  $\lim_{x \rightarrow 2^+} f(x)$

b.  $\lim_{x \rightarrow 2^-} f(x)$

c.  $\lim_{x \rightarrow 2} f(x)$

d.  $\lim_{x \rightarrow 0^-} f(x)$

7. The function  $f(x)$  satisfies the inequality  $2x-7 \leq f(x) \leq \frac{x+4}{2x}$ . Find  $\lim_{x \rightarrow 4} f(x)$ .

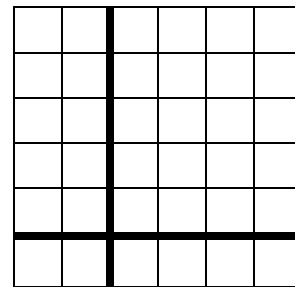
8. (Use your calculator to) estimate the value of the limit:  $\lim_{x \rightarrow 0} (1 - 3x)^{2/x}$ . Express your answer correct to two decimal places.

9. For what value of  $a$  is the function  $f(x)$  continuous on  $(-\infty, \infty)$ ?

$$f(x) = \begin{cases} 4 + x^2 & \text{if } x \geq 2 \\ ax - 2 & \text{if } x < 2 \end{cases}$$

10. Suppose  $f(x)$  is continuous on  $(-\infty, \infty)$  and  $f(2) = -2$ ,  $f(3) = 7$ . What is the name of the theorem we can use to prove that  $f(x)$  has at least one real root between 2 and 3?

11. Sketch (on the right) the graph of a function  $f$  that satisfies the following five conditions:  $f(0) = 1$ ,  $f'(0) = -1$ ,  $f(2) = 0$ ,  $f'(2) = 1$ , and  $f'(1) = 0$ .



12.  $\lim_{x \rightarrow 2} 5x - 9 = 1$ . Finding a number  $\delta$  such that  $|f(x) - 1| < 0.015$  when  $|x - 2| < \delta$ .

13. Prove that  $\lim_{x \rightarrow -5} \left(4 - \frac{3x}{5}\right) = 7$  using the  $\delta \epsilon$  definition of limit. (10 points)

14. Explain why, using the  $\delta \epsilon$  definition of limit, that the function  $H(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$  does not have a limit as  $x$  approaches 0. (Hint:  $\epsilon = 1/2$ .) (Just 1 point)

15. a) Use the definition of the derivative to find the derivative of  $f(x) = 1/x$ . (4 points)

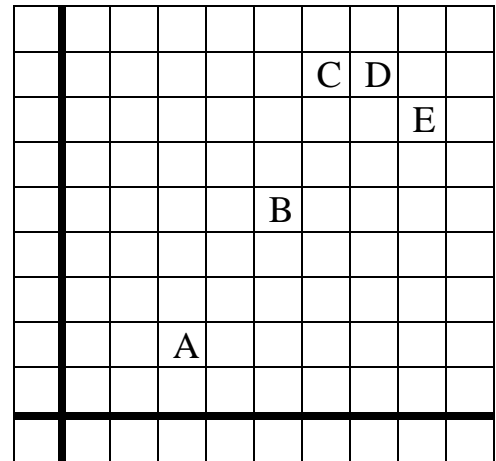
b) Find the equation of the tangent line  $f(x)$  when  $x = 2$ . (4 points)

16. The graph on the right shows the position function of a car. Use the shape of the graph to answer the following. (2 points each)

a. At which point is the car moving the fastest?

b. At which point(s) is the car moving backwards?

c. Between which two points is the car stopped?



d. Was the car speeding up or slowing down at B?