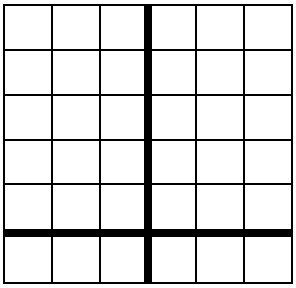
Calculus I (Math 251)
Dr. Caldwell, Fall 2003 **Test One** name (4 points) _____

Relax and enjoy this fun fifty-minute 100-point test covering sections 1.1,4-6 and 2.1-4 of *Calculus Early Transcendentals* by J. Stewart. Clearly indicate your answers––no credit will be given for answers that I cannot find easily. Unless otherwise indicated, all parts of problems are five points.

1. Complete the δ–ε definition of limit: $\lim_{x \to a} f(x) = L$ if for

2. Each part of this problem is worth 3 points.



   a. Sketch the graph of the function
   $$f(x) = \begin{cases} x+2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$$
   b. What would be an appropriate viewing rectangle for the function $500 + \dfrac{x}{x^2 + 100}$? _____ < x < _____; _____ < y < _____ .

   c. What is the domain of $1/(1 - e^x)$?

   d. If $f(x) = 5 + 2x + e^x$, find $f^{-1}(6)$.

   e. What is the exact value of $\log_5 25$?

3. If an arrow is shot upward on the moon with a velocity of 23 m/s, its height in meters after t seconds is $h = 23t - 0.83t^2$.

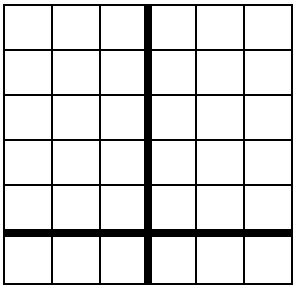
   a. What is the average time velocity over the time period [1,2].

   b. What is the instantaneous velocity at $t = 1$?

4. Sketch (on the right) the graph of a function f that satisfies the following five conditions.

   $f(0) = 2$, $f(2) = 2$, $\lim_{x \to 1} f(x) = \infty$

   $\lim_{x \to 0^+} f(x) = 4$ and $\lim_{x \to 0^-} f(x) = 0$.



5. Evaluate the following limits (or indicate that they do not exist) (4 points each)

   a. $\lim_{x \to 2^-} \sqrt{2 - x}$

   b. $\lim_{x \to 2} \frac{3x - 6}{|x - 2|}$

   c. $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$

   d. $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

   e. $\lim_{x \to -4} \frac{\frac{1}{x} + \frac{1}{4}}{x + 4}$

   f. $\lim_{x \to 1} \frac{x}{(x - 1)^2}$

6. Near the origin (near $x = 0$), the function f(x) satisfies $x - \frac{x^3}{3} \le f(x) \le x + \frac{x^3}{3}$. What is $\lim_{x \to 0} \frac{f(x)}{x}$?

7. (**Use your calculator to**) estimate the value of the limit: $\lim_{x \to 0^+} x^{\sin x}$. Express your answer correct to two decimal places.

8. For what value of a is the function f(x) continuous on $(-\infty,\infty)$?

$$f(x) = \begin{cases} 2x & \text{if } x \ge 1 \\ 2 - ax & \text{if } x < 1 \end{cases}$$

9. Use a graph to find a number δ such that $\frac{x}{(1-x)^2} > 100$ whenever $0 < |\, 1-x \,| < \delta$.

10. $\lim_{x \to 1} 5x - 2 = 3$. Find a number δ such that

$|\, f(x) - 3 \,| < 0.03$ when $0 < |\, x - 1 \,| < \delta$.

11. Explain why, **using the δ–ε definition of limit**, that the floor-function

$\lfloor x \rfloor$ = the greatest integer $\leq x$

does not have a limit as x approaches 3. (2 points)

12. **Prove** that $\lim_{x \to -2} (2x + 5) = 1$ **using** the δ–ε definition of limit. **(10 points)**
(Use the phrases you were told to use!)