

**Test One**

This pleasant fifty-minute test covers parts of chapter two of *Calculus ET* (6E) by Stewart. Clearly indicate your answers--no credit will be given for answers that I cannot find or cannot read. Unless otherwise indicated, all parts of problems are four points each.

- Complete the  $\delta$ - $\epsilon$  definition of limit:  **$\lim_{x \rightarrow a} f(x) = L$  if for**
- The point  $P(1,0)$  lies on the curve  $f(x) = \sin \frac{10\pi}{x}$ . If  $Q$  is the point  $(x, f(x))$ , find the slope of the secant line  $PQ$  (correct to four decimal places) for  $x = 1.1$
- (Use your calculator to) Estimate the value of the limit:  $\lim_{x \rightarrow 0} (1 + 2x)^{1/x}$ . Express your answer correct to four decimal places.
- Sketch (on the right) the graph of a function  $f$  that satisfies the following five conditions.

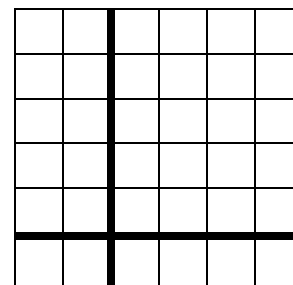
$$f(2) = 3$$

$$f(0) = 3$$

$$\lim_{x \rightarrow 1^+} f(x) = \infty$$

$$\lim_{x \rightarrow 1^-} f(x) = 1$$

$$\lim_{x \rightarrow 2} f(x) = 0$$



5. Evaluate the following limits

a.  $\lim_{x \rightarrow -2^-} \frac{|x+2|}{x+2}$

b.  $\lim_{x \rightarrow \infty} \frac{x^2 - 4}{x^2 + x - 6}$

c.  $\lim_{x \rightarrow 0} \left( \frac{1}{x} + \frac{1}{x-x} \right)$

d.  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + x - 6}$

e.  $\lim_{x \rightarrow -\infty} x^2 + x - 6$

6. Let  $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ e^x & \text{if } 0 \leq x \leq 1 \\ 2 - x & \text{if } x > 1 \end{cases}$

a. Find the numbers at which  $f$  is discontinuous.

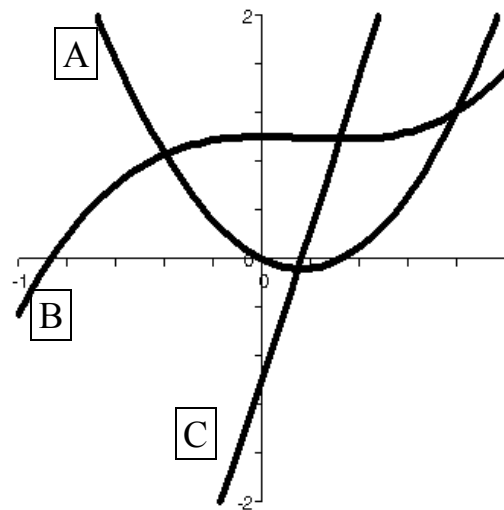
b. At each of these numbers state whether  $f$  is continuous from the right, from the left, or neither.

7. On the right is the graph of three functions. Identify each of the following. (2 points each)

$$f(x) = \underline{\hspace{2cm}}$$

$$f'(x) = \underline{\hspace{2cm}}$$

$$f''(x) = \underline{\hspace{2cm}}$$



8. For what value of  $a$  is the function  $f(x)$  continuous on  $(-\infty, \infty)$ ?

$$f(x) = \begin{cases} 2 + 3x & \text{if } x \geq 1 \\ ax(1+x) & \text{if } x < 1 \end{cases}$$

9. Find the derivative of  $f(x) = 3x^2$  using the limit definition of derivative. (8 points)

9. Prove that  $\lim_{x \rightarrow 2} x - 3 = -1$  by using the  $\delta$ - $\varepsilon$  definition of limit. (6 points)

10. Prove that  $\lim_{x \rightarrow 2} x^2 - 3 = 1$  using the  $\delta$ - $\varepsilon$  definition of limit. (12 points)