Math 251 Fall 2003 ______________________

Test Two name

This taxing but fun fifty-minute test covers sections 2.5 through 3.4 of *Calculus: Early Transcendentals* (5ed) by James Stewart. Clearly [indicate] your answers. Unless otherwise indicated, all parts of problems are four points each.

1. Find the indicated limits.

a. $\lim_{x\to\infty} \left(\sqrt{9x^2 + x} - 3x \right)$

b. $\lim_{x\to -\infty} \frac{\sqrt{9x^2 + x}}{x}$

c. $\lim_{x\to 0} \frac{\sin 5x}{\tan 3x}$

d. $\lim_{x\to 0} \frac{1 - \cos(5x)}{3x}$

2. For what value of a is the function f(x) continuous $(-\infty, \infty)$?

$$f(x) = \begin{cases} 2x & \text{if } x > 5 \\ 3 - ax & \text{if } x < 5 \end{cases}$$

3. A calculus test is thrown in the air and its height in feet is given by $s = 40t - 16t^2$. Find its velocity when $t = 2$.

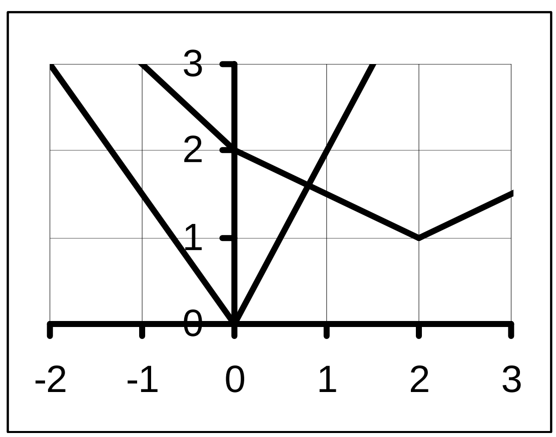
4. Use a limit definition of derivative to show the derivative of $1-3x^2$ is $6x$. (6 points)

5. If f is the focal length of a convex lens and an object is placed at a distance p from the lens, then its image will be at a distance q from the lens, where f, p, and q are related by the lens equation:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

For a fixed focal length f, find the rate of change of q with respect to p. (6 points)

6. Using the graphs on the right, find u'(1) where $u(x) = f(x)g(x)$. (6 points)



7. The curve $y = 1/(1 + x^2)$ is called the witch of Agnesi. Find the equation of the tangent line to this curve at the point (2, 1/5).

8. Find the (first) derivative of the following

a) $x^{51} - 3x^{32} - 23$

b) $x \tan x$

c) $e^x \sin x$

d) $\dfrac{1 - x^2}{1 + x^2}$

e) $4\pi^2$

f) $\cos x \sec x$

g) $\sqrt[3]{t^2} + \sqrt{t^3}$

h) $\sqrt{x}\,(x - 1)$