

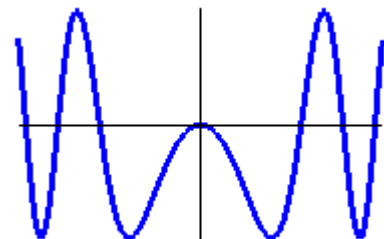
This is our third fun test. Relax and do well. All parts of problems are four points unless otherwise indicated.

1. Find the absolute maximum and absolute minimum of  $f(x) = x^4 - 4x^2 + 2$  on the interval  $[-3, 2]$  (6 points)

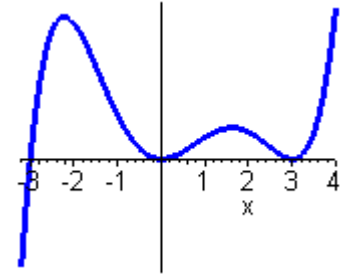
2. Let  $f(x) = x^3 + x - 1$ . Find all of the numbers which satisfy the conclusion of the Mean Value Theorem on the interval  $[0, 2]$ .

3. Show that  $x^5 - 6x + c = 0$  has at most one real zero in the interval  $[-1, 1]$ . (Hint: suppose it has two...)

4. On the right is a graph of the second derivative  $f''$  of a function. How many points of inflection does the original function  $f$  have?



5. On the right a graph of the first derivative  $f'$  of a function is shown.
- a. On what intervals is the function  $f$  increasing?



- b. What are the  $x$ -values of the local extrema of the function  $f$  (if any)?

6. Let  $f(x) = x\sqrt{x+1}$ .
- a. Find the derivative of  $f(x)$ .

- b. Find the critical numbers of  $f(x)$  (if any).

- c. Find the intervals of increase and decrease of  $f(x)$ .

- d. Find the local extrema of  $f(x)$  (if any).

- e. Draw the graph of  $f(x)$ . Label the relative extrema and the intercepts.


7. Let  $y = \frac{1 + 5x^3}{x^3 - x}$ .

a. Find the vertical asymptotes (if any).

b. Find the horizontal asymptotes (if any).

8. Draw a graph of a function  $f(x)$  so that the following six conditions hold.

(6 points)

$f(0) = 0$

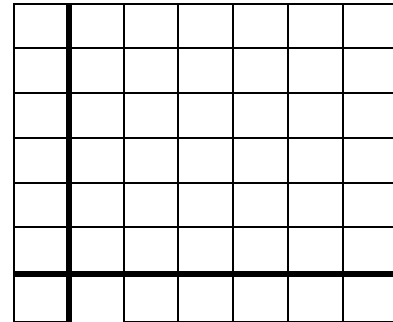
$f(3) = 0$

$f'(1) = 0$

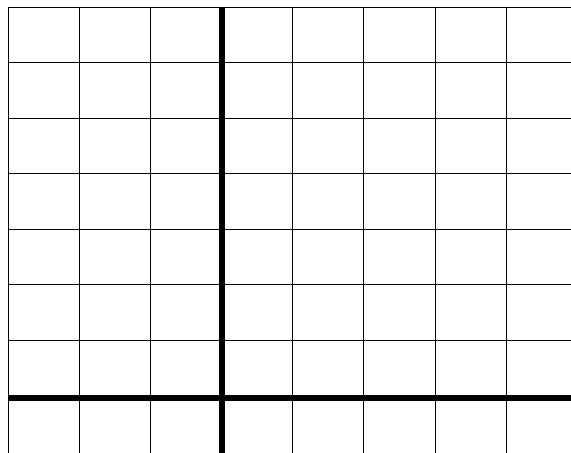
$f'(x) > 0$  if  $x > 2$

$f''(x) > 0$  if  $x < 4$

$\lim_{x \rightarrow \infty} f(x) = 5$



9. Draw the graph of  $f(x) = 4x^4 - 7x^2 - 2x + 5$  and label the extreme values, the inflection points and the asymptotes (if any). (You may approximate to two decimal places.)



10. Find a positive number such that the sum of the number and twice its reciprocal is as small as possible.

11. A rancher wants to fence in 120 square miles with a rectangular fence and then divide it into four equal parts with fences parallel to one side. What dimensions will minimize the amount of fence?

