Math 251 _____

This is our fourth fun test fifty-minute four-page test. It covers section 3.10-11 and 4.1-5 of *Calculus—Early Transcendentals* (6ed) by James Stewart. All parts of problems are four points unless otherwise indicated. Clearly indicate your answers.

1. a) Find the linearization of $f(x) = x^{3/4}$ at $a = 16$.

   b) Use this linearization to approximate $17^{3/4}$.

2. Find the derivative of

   a. $e^{\cosh 3x}$

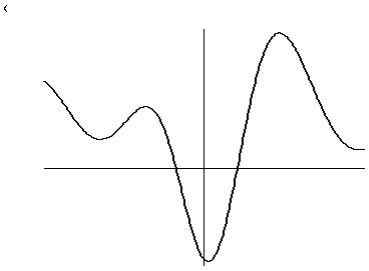
   b. $\tanh^{-1}\sqrt{x}$

3. Find the absolute maximum and absolute minimum of $f(x) = x^4 - 4x^2$ on the interval $[-3,4]$ (6 points)

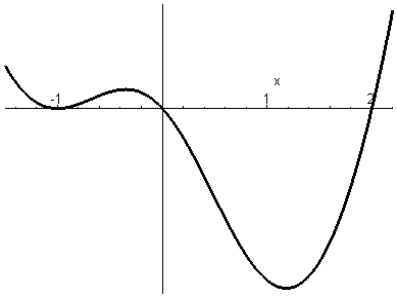
4. Let $f(x) = x^3 + x$. Find all of the numbers that satisfy the conclusion of the Mean Value Theorem on the interval $[-1,1]$.

5. Explain why the MVT shows that the equation $4x + \cos(x) = 0$ has at most one real zeros.

6. On the right is a graph of the second derivative f'' of a function. How many points of inflection does the original function f have?



7. On the right a graph of the first derivative f' of a function is shown.



   a. On what interval(s) is the function f decreasing?

   b. What are the x-values of the local extrema of the function f (if any)?

8. Let $f(x) = \dfrac{x}{x^2 + 1}$.

   a. Find the derivative of f(x).

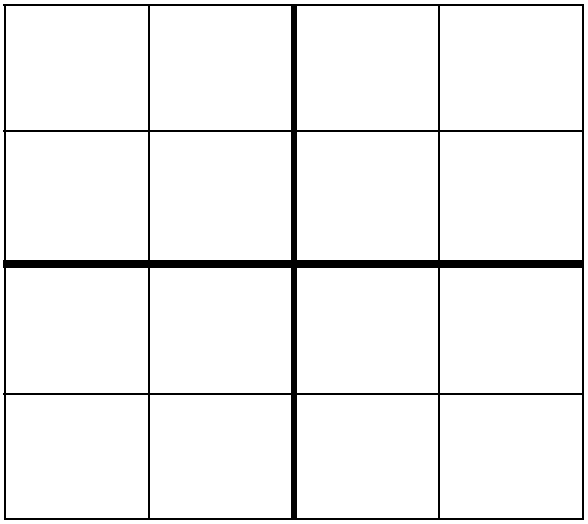
   b. Find the critical numbers of f(x) (if any).

   c. Find the intervals of increase and decrease of f(x).

   d. Find the local extrema of f(x) (if any).

   e. Find any vertical or horizontal asymptotes.

   f. Draw the graph of f(x). Label the relative extrema and the intercepts.



9. a. $\lim_{x \to 0} \frac{x - \sin(x)}{x^3}$

b. $\lim_{x \to \infty} x^{1/x}$

10. Draw a graph of a function f(x) so that the following six conditions hold.

(6 points)

f(0) = –2 f(2) = 0

f’(0) = 0 f’(x) = 0 if $x > 2$

f”(x) > 0 if $x < 1$ $\lim_{x \to 2^-} f(x) = \infty$