

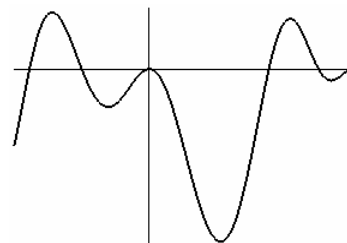
This is our fourth fun test fifty-minute four-page test. It covers chapter four of *Calculus—Early Transcendentals* (5ed) by James Stewart. All parts of problems are four points unless otherwise indicated. Clearly indicate your answers.

1. Find the absolute maximum and absolute minimum of $f(x) = x^3 - 20x^2$ on the interval $[-3, 4]$ (6 points)

2. Let $f(x) = x^2 - 3$. Find all of the numbers that satisfy the conclusion of the Mean Value Theorem on the interval $[0, 1]$.

3. Show that the equation $x^4 + 4x + c = 0$ has at most two real zeros. (Hint: Mean Value Theorem)

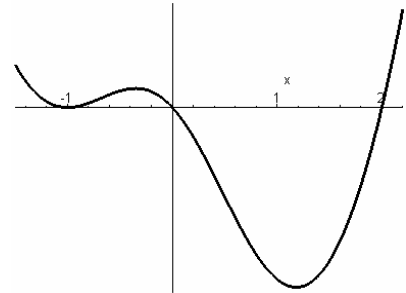
4. On the right is a graph of the second derivative f'' of a function. How many points of inflection does the original function f have? (2 points)



5. On the right a graph of the first derivative f' of a function is shown.

a. On what intervals is the function f increasing?

b. What are the x -values of the local extrema of the function f (if any)?



6. Let $f(x) = (x^2 - 1)^3$.

a. Find the derivative of $f(x)$.

b. Find the critical numbers of $f(x)$ (if any).

c. Find the intervals of increase and decrease of $f(x)$.

d. Find the local extrema of $f(x)$ (if any).

e. Draw the graph of $f(x)$. Label the relative extrema and the intercepts.

7. a. $\lim_{x \rightarrow 0} \frac{5^x - 3^x}{x}$

b. $\lim_{x \rightarrow \infty} \left(1 + \frac{251}{x}\right)^x$

8. Draw a graph of a function $f(x)$ so that the following six conditions hold.

(6 points)

$f(0) = 0$

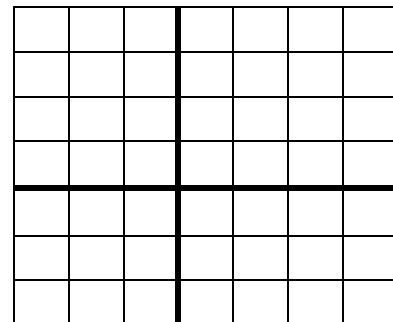
$f(3) = 0$

$f'(2) = 0$

$f'(x) > 0$ if $x > 2$

$f''(x) < 0$ if $x < 1$

$\lim_{x \rightarrow -\infty} f(x) = 2$



9. If 4000 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

10. A football team plays in a stadium that holds 75,000 spectators. With ticket prices at \$10, the average attendance has been 35,000. When ticket prices were lowered to \$8, the average attendance rose to 40,000. Assume the demand function is linear. How should ticket prices be set to maximize attendance?

11. Use Newton's method to determine a solution of $x + \sin(x) = 26$ **correct to eight decimal places**. Show the value you start with, three iterates, and the answer. (8 points)

$x_0 =$ _____
 $x_1 =$ _____
 $x_2 =$ _____
 $x_3 =$ _____
 ...
 $x =$ _____

12. Find $f(x)$ given that $f''(x) = \cos(x)$, $f(0) = 0$ and $f'(0) = 0$. (6 points)

13. Three graphs are shown, which one is the antiderivative of $f(x)$? (2 points)

