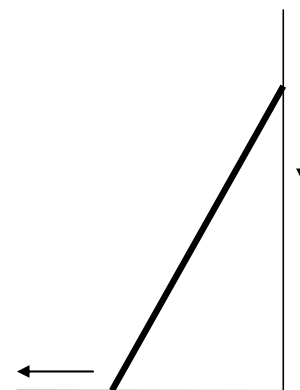


Math 251

This is our fourth test: uplifting, fifty-minute, four-page, 100-point. It covers chapter four of *Calculus—Early Transcendentals* (6ed) by J. Stewart. All parts of all problems are four points unless otherwise indicated. Clearly indicate your answers.

1. Find two positive numbers whose product is 50, and the sum of the first plus twice the second is a minimum. (6 points)

2. Suppose Dr. Caldwell is on the top of a 13 foot ladder cleaning his gutter. His mischievous son Aaron pulls the bottom of the ladder away from the wall at a rate of 1 foot per second. How fast is the top of the ladder descending when the bottom of the ladder is 5 feet from the wall. (8 points)



3. Evaluate the integral $\int_{-4}^4 \sqrt{16 - x^2} dx$ by interpreting it in terms of an area.

4. Use Newton's method to determine the next 4 approximations to $\sqrt[7]{251}$ (Hint: view this as $x^7 - 251 = 0$). Give the approximations correct to 7 decimal places. (2 points each)

$$x_1 = 2.0000000$$

$$x_2 = \underline{\hspace{10em}}$$

$$x_3 = \underline{\hspace{10em}}$$

$$x_4 = \underline{\hspace{10em}}$$

$$x_5 = \underline{\hspace{10em}}$$

5. Find $f(x)$ for each of the following. (6 points each)

a) $f'(x) = 4x - x^4$, $f(1) = 0$.

b) $f''(x) = 60x^2$, $f(0) = 0$, $f'(0) = 5$.

6. Find the derivative of the function $h(x) = \int_{-4}^{x^2} \sqrt{16 - y^2} dy$.

7. Use five rectangles and left endpoints to approximate the area under $y = x^2$ from $x = 0$ to $x = 5$. (8 points)

8. Evaluate the following integrals.

a. $\int_0^4 \sqrt{x} \, dx$

b. $\int_1^4 \frac{x^2 - 1}{x} \, dx$

c. $\int_0^{\pi/4} \sec \theta \tan \theta \, d\theta$

d. $\int_1^4 (x - 1)(x + 1) \, dx$

9. Evaluate the following integrals.

(5 points each)

a. $\int \frac{x^2 - 1}{x^4 - 1} dx$

b. $\int e^x + \cosh x dx$

c. $\int \frac{(\ln x)^{12}}{x} dx$

d. $\int_0^{\pi/4} \sec^2 \theta \tan^9 \theta d\theta$

e. $\int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx$

f. $\int_{-\pi/4}^{\pi/4} \sin^{251} x dx$