Calculus I, Math 251
Last One-hour Test

_____
name (10 points)

This heart warming fifty-minute test "covers" chapters six and nine of *Calculus* by Stewart. Clearly indicate your answers. Except where indicated, each part of each problem is worth five points. Pay attention to whether or not you need to evaluate the integrals.

1. Find the area between the curves: $y = \sec^2 x$, $y = 0$, $x = 0$ and $x = \pi/4$.

   a. The integral:

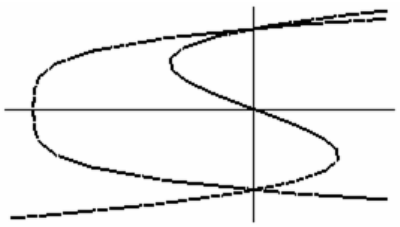
   b. The exact answer:

   c. What is the average value of $\sec^2 x$ on this interval?

2. Find the area between the curves: $x = y^3 - y$ and $x = y^4 - 1$.

   a. The integral:

   b. The exact answer:



3. The base of an object S is the area between $y = 4 - x^2$ and the x-axis. Cross-sections perpendicular to the x-axis are circles. Set up an integral to evaluate the volume of the solid S. (Do not evaluate it.)

4. A swimming pool is 10 m wide, 100 m long and 2 m deep is filled to a depth of 1 m with fresh water (density 1000 kg/m^3). Find the hydrostatic force on one end of the pool.

   a. The integral:

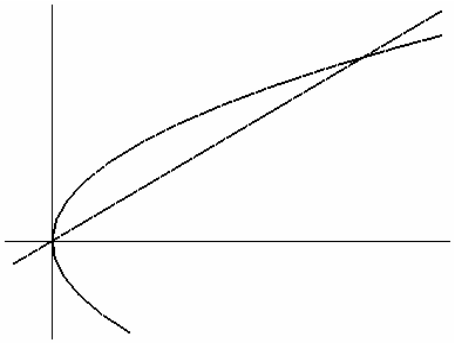
   b. An **approximate** answer (e.g., you may use your calculator):

5. Find the volume of the solid obtained by rotating the region bounded by $y = x^2 + x - 2$ and $y = 0$ about the y–axis. (10 points)

   The integral (you do not need to evaluate it):

6. Find the volume of the solid obtained by rotating the region bounded by $y^2 = x$, and $x = 2y$ about the x–axis. (10 points)

   The integral (you do not need to evaluate it):



7. A spring has a natural length of 10 cm. A 20-N force is required to stretch it from its natural length to a length of 20 cm..

   a. Find the spring constant k.

   b. Find exactly how much work is required to stretch the spring from 20 to 40 cm.

8. Find the arc-length of the curve $y = \frac{2}{3}(x^2-1)^{3/2}$ from $x = 1$ to $x = 3$.
   a. The integral (Hint: the term under the radical should be a perfect square):

   b. The exact answer. (**Show your work**!)

10. Find the surface area obtained by rotating the curve $y = \sqrt{4 - x^2}$ $(0 \leq x \leq 1)$ about the x-axis.

    a. Find the derivative (dy/dx)

    b. Set up the integral.