

This easy fifty minute test covers sections 8.8, 10.1,4,5,7 and 11.1,2 of *Calculus* by James Stewart 4th ed. Clearly indicate your answers and **show your work**. All parts of problems are five points each (unless otherwise stated).

1. Evaluate the following improper integrals.

a) $\int_1^{\infty} 6x^2 e^{-x^3} dx$

b) $\int_0^6 \frac{1}{x\sqrt{x}} dx$

2. For which values of k does the function $y = e^{kt}$ satisfy the differential equation $y'' - 3y' - 10y = 0$.

3. A bacteria culture starts with 2 bacteria and grows at a rate proportional to its size. After 30 minutes the culture contains 128 bacteria.

a) Find an expression for the number of bacterial after t minutes.

b) What will be the population after 60 minutes?

c) When (after how many minutes) will the population reach 1,000,000.

4. Suppose populations of mathematics teachers M and students S are modeled by the equations

$$\begin{cases} \frac{dS}{dt} = 32S - 2SM \\ \frac{dM}{dt} = -80M + 0.1SM \end{cases}$$

a) Find the equilibrium point(s)

b) This is a classical predator-prey system, which is the predator? (1 point)

d) Find dS/dM .

c) Below is the direction field for this equation. Sketch a solution (trajectory) for a population starting at $(M,S) = (6,400)$. (Hint: Use your knowledge of the equilibrium points to figure out the scale in the graph.)



