Math 252 – Spring 2001

Relax and enjoy this thrilling test on sections 12.4 through 12.11 of *Calculus* by James Stewart. Error estimates and applications (12.12 plus …) will be covered on the final.

1. Test **five of the following six series** for convergence. State whether they are

   (a) absolutely convergent, (b) conditionally convergent, or (c) divergent.

   If you want full credit, show enough of your work to indicate which tests you are using. For example, show the limit if you are using a limit test… (4 points each)

a) $\sum_{n=1}^{\infty} k^{-1.001}$

b) $\sum_{n=1}^{\infty} (-1)^n \frac{n-1}{n^2+1}$

c) $\sum_{n=1}^{\infty} \left(\frac{2n}{n+3} \right)^{n^2}$

d) $\sum_{n=1}^{\infty} \frac{(-2)^{3n}}{(2n)^n}$

e) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n-1)}$

f) $\sum_{n=1}^{\infty} (-1)^n \frac{n!}{2^n}$

2. Find the Maclaurin series for the following.

a) $\dfrac{1}{1+x^2}$ (6 points each)

b) $\tan^{-1}(x^2)$ (Hint: what is $\dfrac{d}{dx}\tan^{-1}x$?)

c) $\int \tan^{-1}(x^2)dx$

d) What is the radius of convergence of the above series (they are all the same). (3 points)

3. Give the first four terms of the Taylor series for (6 points each)

a) x^2e^x

b) $\sin x^4$

4. a) Give the first five terms of the expansion for $\sqrt[3]{1+3x}$. (10 points)

b). What is the radius of convergence of the series expansion you found above? (3 points)

5. Find the Taylor polynomial of degree six, T_6, for $\sin x$ about the point $x = \pi/2$. (12 points)