

TEN BASIC RULES OF INFERENCE

Negation Introduction (\sim I – indirect proof IP)

Assume p
Get q & $\sim q$
 $\vdash \sim p$

Negation Elimination (\sim E – version of DN)

$\sim\sim p \rightarrow p$

Conditional Introduction (\rightarrow I – conditional proof CP)

Assume p
Get q
 $\vdash p \rightarrow q$

Conditional Elimination (\rightarrow E – modus ponens MP)

$p \rightarrow q$
 p
 $\vdash q$

Conjunction Introduction ($\&$ I – conjunction CONJ)

p
 q
 $\vdash p \& q$

Conjunction Elimination ($\&$ E – simplification SIMP)

$p \& q$
 $\vdash p$

Disjunction Introduction (\vee I – addition ADD)

p
 $\vdash p \vee q$

Disjunction Elimination (\vee E – version of CD)

$p \vee q$
 $p \rightarrow r$
 $q \rightarrow r$
 $\vdash r$

Biconditional Introduction (\leftrightarrow I – version of ME)

$p \rightarrow q$
 $q \rightarrow p$
 $\vdash p \leftrightarrow q$

Biconditional Elimination (\leftrightarrow E – version of ME)

$p \leftrightarrow q$
 $\vdash p \rightarrow q$
or
 $\vdash q \rightarrow p$

IMPORTANT DERIVED RULES OF INFERENCE

Modus Tollens (MT)

$p \rightarrow q$
 $\sim q$
 $\vdash \sim p$

Hypothetical Syllogism (HS)

$p \rightarrow q$
 $q \rightarrow r$
 $\vdash p \rightarrow r$

Disjunctive Syllogism (DS)

$p \vee q$
 $\sim p$
 $\vdash q$

Absorption (ABS)

$p \rightarrow q$
 $\vdash p \rightarrow (p \& q)$

Constructive Dilemma (CD)

$p \vee q$
 $p \rightarrow r$
 $q \rightarrow s$
 $\vdash r \vee s$

Repeat (RE)

p
 $\vdash p$

Contradiction (CON)

p
 $\sim p$
 \vdash Any wff

EQUIVALENCES

De Morgan's Law (DM)

$\sim(p \& q) \leftrightarrow (\sim p \vee \sim q)$
 $\sim(p \vee q) \leftrightarrow (\sim p \& \sim q)$

Commutation (COM)

$(p \vee q) \leftrightarrow (q \vee p)$
 $(p \& q) \leftrightarrow (q \& p)$

Association (ASSOC)

$[p \vee (q \vee r)] \leftrightarrow [(p \vee q) \vee r]$
 $[p \& (q \& r)] \leftrightarrow [(p \& q) \& r]$

Distribution (DIST)

$[p \& (q \vee r)] \leftrightarrow [(p \& q) \vee (p \& r)]$
 $[p \vee (q \& r)] \leftrightarrow [(p \vee q) \& (p \vee r)]$

Double Negation (DN)

$p \leftrightarrow \sim\sim p$

Transposition (TRANS)

$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

Material implication (MI)

$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$

Material Equivalence (ME)

$(p \leftrightarrow q) \leftrightarrow [(p \& q) \vee (\sim p \& \sim q)]$
 $(p \leftrightarrow q) \leftrightarrow [(p \rightarrow q) \& (q \rightarrow p)]$

Exportation (EXP)

$[(p \& q) \rightarrow r] \leftrightarrow (p \rightarrow (q \rightarrow r))$

Tautology (TAUT)

$p \leftrightarrow (p \& p)$
 $p \leftrightarrow (p \vee p)$