Math 110, Exam I

There are 11 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. There are 130 points available on this exam.

1. Sketch the graph of the following polynomial function. Show all x-intercepts and the behavior of the graph at each and show the left- and right-hand behavior. (6 points)

   \[ A(x) = (x - 4)^2(x - 8)(x + 5)^2 \]

2. Determine a polynomial in factored form whose graph would look like the graph shown below where the y-intercept is located at (0, 2). (6 points)

   \[ y = \frac{1}{50} (x+5)^2(x - \frac{1}{2})(x - 8) \]
3. For each of the following functions indicate by letter which of the following describes the left- and right-hand behavior of the function’s graph. (3 points each)

(A) \[ y = -3x^4 + 5x^3 - 2x^2 + 5x - 7 \]
(B) \[ y = 5x^3 - 6x + 12 \]
(C) \[ y = (2x - 1)^2(x + 8)(x - 4) \]
(D) \[ y = (1 - x)^3(4 + x)^2 \]

4. The nonreal number \(1 + 2i\) is a zero of the following polynomial. Determine all zeros of this polynomial and show its complete factorization. (8 points)

\[ B(x) = x^4 - 2x^3 + 14x^2 - 18x + 45 \]

Zeros: \(1 + 2i, 1 - 2i, 3i, -3i\)

Factorization:
\[
(X - 1 - 2i)(X - 1 + 2i)(X - 3i)(X + 3i)
\]
5. Let \( C(x) = 2x^4 - 3x^3 - 15x^2 + 15x + 25. \)

(a) Use Descartes’ rule of signs to predict the number of positive real zeros of \( C(x) \). (3 pts)
\[ + - - + \]
2 or 0

(b) Use Descartes’ rule of signs to predict the number of negative real zeros of \( C(x) \). (3 pts)
\[ C(-x) = 2x^4 + 3x^3 - 15x^2 - 15x + 25 \]
\[ + - - + \]
2 or 0

(c) List the possible rational zeros of \( C(x) \). (6 points)
\[ \pm 1, \pm 5, \pm 25 \]
\[ \pm 1, \pm \frac{1}{2}, \pm 5, \pm \frac{5}{2}, \pm 25, \pm \frac{25}{2} \]
\[ \pm 1, \pm 2 \]

(d) Use your calculator to find two rational zeros of \( C(x) \). (4 points)
\[ -1, \frac{5}{2} \]

(e) Use synthetic division to “divide out” the rational zeros found in part (d) to reduce from a fourth degree polynomial to a quadratic. (6 points)

\[ \begin{array}{c|ccccc}
-1 & 2 & -3 & -15 & 15 & 25 \\
\hline
& -2 & 5 & 10 & -25 \\
\end{array} \]

\[ \begin{array}{c|ccccc}
\frac{5}{2} & 2 & -5 & -10 & 25 & 0 \\
\hline
& 2 & 5 & 0 & -25 \\
\end{array} \]

(f) What factorization does the result of part (e) give you for \( C(x) \)? (4 points)
\[ (x + 1)(x - \frac{5}{2})(2x^2 - 10) \]

(g) Determine the other two zeros of \( C(x) \). (4 points)
\[ \pm \sqrt{5} \]
6. Consider the quadratic function \( f(x) = -2x^2 - 12x + 5 \). Complete the following statements. (2 points per blank)

The graph of \( y = f(x) \) is a parabola that opens \( \underline{\text{down}} \) and has the point \( (-3, \frac{23}{2}) \) as its vertex. This vertex is a \( \underline{\text{maximum}} \).

The domain of the function \( f(x) \) is \( (-\infty, \infty) \) and the range of the function \( f(x) \) is \( (-\infty, \frac{23}{2}] \). When written in shifted form, \( f(x) = -2(x+3)^2 + \frac{23}{2} \).

The x-intercepts on the graph of \( y = f(x) \) are \( (-3 - \sqrt{\frac{23}{2}}, 0) \) and \( (-3 + \sqrt{\frac{23}{2}}, 0) \).

The y-intercept on the graph of \( y = f(x) \) is \( (0, \frac{5}{4}) \).

The graph of \( y = f(x) \) is the graph of \( y = -2x^2 \) shifted \( 3 \) units \( \underline{\text{left}} \) and then shifted \( \frac{23}{2} \) units \( \underline{\text{up}} \).

\[
-2(x+3)^2 + \frac{23}{2} = 0
\]
\[
23 = 2(x+3)^2
\]
\[
\frac{23}{2} = (x+3)^2
\]
\[
\pm \sqrt{\frac{23}{2}} = x+3
\]
\[
-3 \pm \sqrt{\frac{23}{2}} = x
\]
8. Determine all zeros of \( g(x) = x^4 - x^3 - 33x^2 + 56x + 12 \). (12 points)

\[
\begin{array}{cccc}
\text{zeros:} & -6 & 2 & \frac{5 - \sqrt{29}}{2}, \frac{5 + \sqrt{29}}{2} \\
-6 & 1 & -5 & 2 & 10 \\
-2 & 1 & -5 & -1 & 0 \\
& & x^2 - 5x - 1 & \quad x = \frac{5 + \sqrt{25 + 4}}{2}
\end{array}
\]

9. Let \( h(x) = 2x^4 - 7x^3 + 7x^2 + x - 4 \).

(a) Determine an integer upper bound on the zeros of \( h(x) \). Show the necessary synthetic division calculation. (4 points)

\[
\begin{array}{cccc}
4 & 2 & -7 & 7 & 1 & -4 \\
& 8 & 4 & 4 & & \\
& 2 & 1 & 1 & 5 & \\
4 & & & & &
\end{array}
\]

(b) Determine an integer lower bound on the zeros of \( h(x) \). Show the necessary synthetic division calculation. (4 points)

\[
\begin{array}{cccc}
-1 & 2 & -7 & 7 & 1 & -4 \\
& -2 & 9 & -16 & -15 & \boxed{-1} \\
& 2 & -9 & 16 & -15 & 11 \\
\end{array}
\]

(c) Since \( h(1) = -1 \) and \( h(2) = 2 \) we know \( h(x) \) has a zero between \( x = 1 \) and \( x = 2 \) because of what theorem? (3 points)

\text{Intermediate Value Theorem}

(d) Use your calculator to approximate the zero described in part (c) to the nearest thousandth. (4 points)

\[1.618\]
10. An open box is made by cutting four equal squares from the corners of a rectangular piece of cardboard that is 11 inches by 35 inches and folding up the sides.

(a) What polynomial gives the volume of the box if the side of the square cut out has length x? (4 points)

\[ V = x(11-2x)(35-2x) \]

(b) What are the dimensions of the box with maximum volume? (4 points)

\[ x = 2.5 \]

\[ 2.5'' \times 6'' \times 30'' \]

11. Is the following data better modeled with a quadratic or a cubic polynomial? Explain your answer with appropriate reference to an \( r^2 \)-value. (5 points)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>45</td>
</tr>
<tr>
<td>12</td>
<td>52</td>
</tr>
<tr>
<td>18</td>
<td>61</td>
</tr>
<tr>
<td>24</td>
<td>58</td>
</tr>
<tr>
<td>30</td>
<td>49</td>
</tr>
<tr>
<td>32</td>
<td>51</td>
</tr>
<tr>
<td>35</td>
<td>46</td>
</tr>
</tbody>
</table>

For quadratic
\[ r^2 = 0.8151 \]

For cubic
\[ r^2 = 0.9567 \]

So cubic is better