Math 251, Exam I Name ___________________________

There are 15 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. Unless specified otherwise all answers should be exact.

In problems 1-8 evaluate the given limits as a number L, +∞, or –∞. (4 points each)

1. $\lim_{x\to 3}\left(3-\sqrt{5x+1}\right)$

$= 3 - \sqrt{15+1}$

$= 3 - 4$

$= -1$

2. $\lim_{x\to 2}\left(\frac{3x-6}{x^2-5x+6}\right)$

$= \lim_{x\to 2}\left(\frac{3\cancel{(x-2)}}{\cancel{(x-2)}(x-3)}\right)$

$= \frac{3}{-1} = -3$

3. $\lim_{x\to\infty}\left(\frac{2x+5}{3x-1}\right)$

$= \frac{2}{3}$

4. $\lim_{x\to 0}\left(\frac{\sin(2x)}{x}\right)$

$= 2$

5. $\lim_{x\to 4^+}\left(\frac{2x}{x-4}\right)$

$= +\infty$

6. $\lim_{x\to 0}\left(\frac{2}{\cos(x)-1}\right)$

$= -\infty$

7. $\lim_{x\to 3^-}\left(2x^2-3x\right)$

$= 2(3)^2 - 3\cdot 3$

$= 9$

8. $\lim_{x\to 2}\left(\tan\left(\frac{\pi}{2x}\right)\right)$

$= \tan\frac{\pi}{4}$

$= 1$

9. Let $f(x)=\begin{cases}\sqrt{7-x} & if\ x<3\\ 2x-5 & if\ 3<x\le 5\\ 10-x & if\ x>5\end{cases}$

Determine the following limits. If a particular limit does not exist write DNE. (2 points each)

(a) $\lim_{x\to 5^-} f(x)$

$= \lim_{x\to 5^-}(2x-5)$

$= 5$

(b) $\lim_{x\to 5^+} f(x)$

$= \lim_{x\to 5^+}(10-x)$

$= 5$

(c) $\lim_{x\to 5} f(x)$

$= 5$

(d) $\lim_{x\to 3^-} f(x)$

$= \lim_{x\to 3^-}(\sqrt{7-x})$

$= 2$

(e) $\lim_{x\to 3^+} f(x)$

$= \lim_{x\to 3^+}(2x-5)$

$= 1$

(f) $\lim_{x\to 3} f(x)$

DNE

(g) $\lim_{x\to 4} f(x)$

$= \lim_{x\to 4}(2x-5)$

$= 3$

(h) $\lim_{x\to 8} f(x)$

$= \lim_{x\to 8}(10-x)$

$= 2$

Recall that when we verify $\lim_{x\to a} f(x) = L$, for a given ε > 0 we must find a number δ > 0 so that $|f(x) - L| < \varepsilon$ whenever x satisfies $0 < |x - a| < \delta$.

10. In verifying that $\lim_{x\to 4}(5x+2) = 22$ if we take $\varepsilon = 0.08$, what is the best possible choice for δ? Show all necessary work. (5 points)

$19.92 < 5x+2 < 22.08$

$17.92 < 5x < 20.08$

$3.584 < x < 4.016$

Take $\delta = 0.016$

$1.9 < \sqrt{x+1} < 2.1$
$3.61 < x+1 < 4.41$
$2.61 < x < 3.41$

11. In verifying that $\lim_{x \to 3} \sqrt{x+1} = 2$ if we take $\varepsilon = 0.1$, which of the following values of δ could we use? Circle all that could be used. (5 points)

0.42 0.41 0.40 0.39 0.38 0.37 0.36

12. Let $g(x) = 2x^2 - 1$.

(a) Determine the slope of the secant line through the points (x, g(x)) and (3, g(3)) where $x \neq 3$. Simplify your answer (3 points)

$$\frac{2x^2-1-17}{x-3} = \frac{2x^2-18}{x-3} = \frac{2(x^2-9)}{x-3}$$

$$= 2(x+3), \quad x \neq 3$$

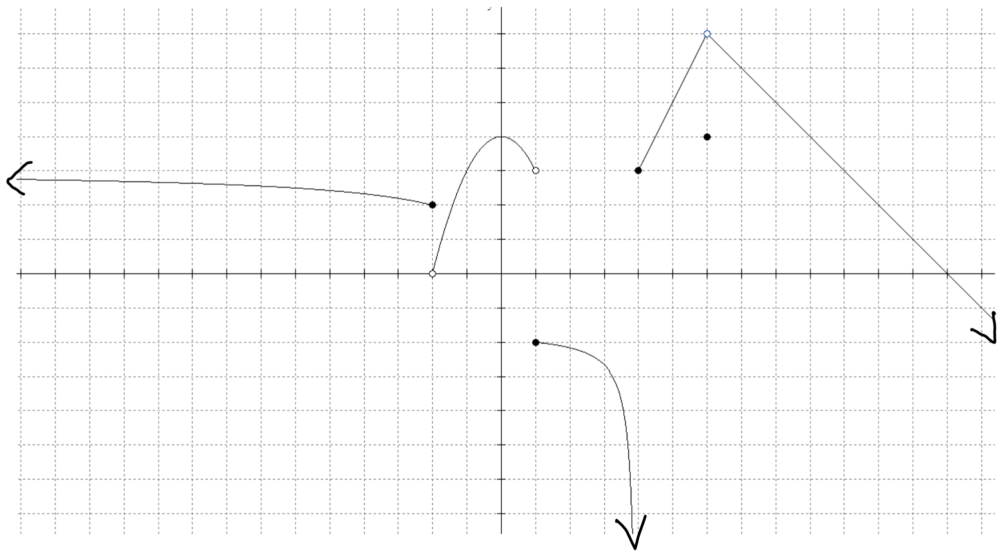
(b) Complete the following table of values. If necessary round all values to four decimal places. (3 points)

x	Slope of secant line through (x, g(x)) and (3, g(3))
2.9	11.8
2.99	11.98
2.999	11.998
3.1	12.2
3.01	12.02
3.001	12.002

(c) Based on your answers in part (b), what is the best guess for the slope of the tangent line to the graph of y = g(x) at the point where x = 3? (2 points)

12

13. Use the graph of the function *h(x)* shown below to determine the following limits. If a particular limit does not exist write DNE. (3 points each)



(a) $\lim_{x \to -\infty} h(x)$

3

(b) $\lim_{x \to -2^-} h(x)$

2

(c) $\lim_{x \to -2^+} h(x)$

0

(d) $\lim_{x \to -2} h(x)$

DNE

(e) $\lim_{x \to 6^-} h(x)$

7

(f) $\lim_{x \to 6^+} h(x)$

7

(g) $\lim_{x \to 6} h(x)$

7

14. List the discontinuities for the function whose graph is shown above and identify each as removable, jump, or infinite. (8 points)

$x = -2$ jump

$x = 1$ jump

$x = 4$ infinite

$x = 6$ removable

15. Determine the intervals of continuity for the function whose graph is shown above. (5 points)

$(-\infty, -2] \cup (-2, 1) \cup [1, 4) \cup [4, 6) \cup (6, \infty)$