There are 20 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. Unless specified otherwise, all answers should be exact. Each problem is worth 5 points unless indicated otherwise.

Use the three vectors given below in problems 1-10.

\( \vec{u} = (3, -3, 2) \quad \vec{v} = (-5, 5, 6) \quad \vec{w} = (1, 2, 4) \)

1. Determine \( \vec{u} \cdot \vec{v} \).

\[
-15 - 15 + 12 = -18
\]

2. Determine \( \vec{u} \times \vec{v} \).

\[
\begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
3 & -3 & 2 \\
-5 & 5 & 6
\end{vmatrix} = (-28, -28, 0)
\]

3. Determine \( |\vec{w}| \)

\[
\sqrt{1 + 4 + 16} = \sqrt{21}
\]

4. Determine the angle between \( \vec{u} \) and \( \vec{v} \).

Round your answer to the nearest tenth of a degree.

\[
\cos \theta = \frac{-18}{\sqrt{22 \cdot \sqrt{86}}}
\]

\[
\theta \approx 114.4^\circ
\]

5. Determine the equation of the plane containing the points \((0, 0, 0)\), \((3, -3, 2)\), and \((-5, 5, 6)\).

\[-28x - 28y + 0z = 0\]

6. Determine the equation of a line containing the point \((3, 8, -3)\) that is parallel to \( \vec{w} \)

\[
\left< 3 + t, 8 + 2t, -3 + 4t \right>
\]

7. Determine the point that is 15 units from the point \((3, 8, -3)\) in the direction of \( \vec{w} \).

\[
\left< 3, 8, -3 \right> + \frac{15}{\sqrt{21}} \left< 1, 2, 4 \right> = \left< 3 + \frac{15}{\sqrt{21}}, 8 + \frac{30}{\sqrt{21}}, -3 + \frac{60}{\sqrt{21}} \right>
\]
\[ \vec{u} = (3, -3, 2) \quad \vec{v} = (-5, 5, 6) \quad \vec{w} = (1, 2, 4) \]

8. Determine the equation of a line containing the point \((3, 8, -3)\) that is orthogonal to \(\vec{w}\)

\[ \langle 3 + at, 8 + bt, -3 + ct \rangle \text{ where } a + 2b + 4c = 0 \]

9. Determine the projection of \(\vec{v}\) along \(\vec{u}\).

\[ \frac{-18}{22} \langle 3, -3, 2 \rangle = \langle \frac{-27}{11}, \frac{27}{11}, \frac{-18}{11} \rangle \]

10. Write \(\vec{v}\) as the sum of two vectors, one that is parallel to \(\vec{u}\) and one that is orthogonal to \(\vec{u}\)

\[ \langle -5, 5, 6 \rangle = \langle \frac{-27}{11}, \frac{27}{11}, \frac{-18}{11} \rangle + \langle \frac{-28}{11}, \frac{28}{11}, \frac{84}{11} \rangle \]

Use the following points, lines, and planes to work problems 11-20.

\(P_1 : 5x - 5y + 4z = 6\) \quad \(P_2 : x + 2y + 7z = 4\) \quad \(P_3 : -15x + 15y - 12z = 5\)

\(\ell_1 : x = 2t + 8, y = 3t + 4, z = 7t + 5\) \quad \(\ell_2 : x = 4t - 14, y = 2t - 1, z = 6t - 16\)

\(U = (5, 1, -2)\) \quad \(V = (5, -4, 4)\) \quad \(W = (6, 7, 2)\)

11. Which two of these planes are parallel?

\[ P_1 \parallel P_3 \]

12. Determine a plane that is orthogonal to \(P_1\) and contains the point \(U\).

\[ a(x - 5) + b(y - 1) + c(z + 2) = 0 \]

where \(5a - 5b + 4c = 0\)
\( P_1 : 5x - 5y + 4z = 6 \quad P_2 : x + 2y + 7z = 4 \quad P_3 : -15x + 15y - 12z = 5 \)

\[ \ell_1 : x = 2t + 8, y = 3t + 4, z = 7t + 5 \quad \ell_2 : x = 4t - 14, y = 2t - 1, z = 6t - 16 \]

\[ U = (5, 1, -2) \quad V = (5, -4, 4) \quad W = (6, 7, 2) \]

13. Determine the distance between points \( U \) and \( V \).

\[ \sqrt{(5-5)^2 + (-4-1)^2 + (4+2)^2} = \sqrt{61} \]

14. Determine the acute angle between the planes \( P_1 \) and \( P_2 \). Round your answer to the nearest tenth of a degree.

\[ \cos \theta = \frac{\left| \frac{5 - 10 + 28}{\sqrt{66}} \right|}{\sqrt{54}} \quad \theta \approx 67.3^\circ \]

15. Determine the equation of the line containing the point \( U \) that is parallel to the line through the points \( V \) and \( W \).

\[ \langle 5 + t, 1 + 11t, -2 - 2t \rangle \]

16. Determine the distance from \( U \) to \( P_2 \).

\[
\text{Point in } P_2 \ (0, 2, 0) \\
\langle 5, -1, -2 \rangle \\
\left| \frac{5 - 2 - 14}{\sqrt{54}} \right| = \frac{11}{\sqrt{54}}
\]
17. Determine the equation of the line that is the intersection of planes $P_1$ and $P_2$.

\[ \frac{32}{15} - \frac{43}{15} t, \frac{14}{15} - \frac{31}{15} t, t \]

18. Determine the point(s) at which $P_1$ intersects $\ell_1$.

\[ 5(2t+8) - 5(3t+4) + 4(7t+5) = 6 \]
\[ 23t = -34 \]
\[ t = -\frac{34}{23} \]
\[ \left( \frac{116}{23}, \frac{-10}{23}, \frac{-123}{23} \right) \]

19. Determine the equation of the sphere centered at $U$ with a radius of 4 units.

\[ (x-5)^2 + (y-1)^2 + (z+2)^2 = 16 \]

20. Determine the equation of the plane containing $U$ and $\ell_1$.

Point on $\ell_1$, $(8,4,5)$

\[ \begin{vmatrix} i & j & k \\ 3 & 3 & 7 \\ 2 & 3 & 7 \end{vmatrix} = \langle 0, -7, 3 \rangle \]

-7y + 3z = -13

5-Point Bonus: Determine the point of intersection of lines $\ell_1$ and $\ell_2$.

\[ (14, 13, 26) \]