1. Let \( f(x) = \frac{2x^2 - 8x}{x^2 - 4x - 21} = \frac{2x(x - 4)}{(x - 7)(x + 3)} \).

(a) Determine the domain of \( f(x) \). (3 points)

\[ x \neq 7, -3 \]

(b) Determine the vertical asymptote(s) on the graph of \( y = f(x) \). (3 points)

\[ x = 7, \quad x = -3 \]

(c) Determine the horizontal asymptote on the graph of \( y = f(x) \). (3 points)

\[ y = 2 \]

(d) Determine the \( y \)-intercept on the graph of \( y = f(x) \). (3 points)

\[ (0, 0) \]

(e) Determine the \( x \)-intercept(s) on the graph of \( y = f(x) \). (3 points)

\[ (0, 0), (4, 0) \]

(f) Sketch the graph of \( y = f(x) \) showing all asymptotes and \( x \)-intercepts. (5 points)
2. Sketch the graph of \( g(x) = \frac{(2x + 1)(x - 2)^2(x - 5)}{(x - 8)^2(x + 5)(x - 1)} \). Show all asymptotes and x-intercepts. (8 points)
3. Let \( h(x) = 2 + \frac{1}{x} \).

(a) Determine \( h^{-1}(0) \). (3 points)

\[ h^{-1}(0) = -\frac{1}{2} \]

(b) Determine \( h^{-1}(x) \). (3 points)

\[
\begin{align*}
y &= 2 + \frac{1}{x} \\
x &= \frac{1}{y-2} \\
y - 2 &= \frac{1}{x} \\
h^{-1}(x) &= \frac{x}{x-2}
\end{align*}
\]

(c) Determine \( (h \circ h^{-1})(6) \). (3 points)

\[ (h \circ h^{-1})(6) = 6 \]

(d) Determine the range of \( h(x) \). (3 points)

\[ \text{Range of } h = \text{Domain of } h^{-1}: \quad x \neq 2 \]

(e) Determine the range of \( h^{-1}(x) \). (3 points)

\[ \text{Range of } h^{-1} = \text{Domain of } h: \quad y \neq 0 \]
4. The graph of a one-to-one function $P(x)$ is shown below.

(a) Determine $P(-2)$. (3 points)

(b) Determine $P^{-1}(-2)$. (3 points)

(c) Determine the domain of $P^{-1}(x)$. (3 points)

(d) Determine the range of $P^{-1}(x)$. (3 points)

(e) On the graph with $P$ draw the graph of $y = P^{-1}(x)$. (3 points)
5. Match the following rational functions with their descriptions given below. Each question has one correct answer. However, the functions may be used more than once. (3 points each)

\[ A(x) = \frac{x^2 + x}{9 - x^2} \quad B(x) = \frac{(x + 1)(x - 3)^2}{x + 9} \quad C(x) = \frac{1 - x^3}{x^2 - 9} \]

\[ D(x) = \frac{x + 2}{(x + 1)(x - 3)^2} \quad E(x) = \frac{2x^3 - 4}{2x^2 - 8} \quad F(x) = \frac{x^2 - 1}{x^2 + 5x} \]

(a) Has the line \( x = 2 \) as an asymptote

(b) Has no \( y \)-intercept

(c) Has the \( x \)-axis as an asymptote

(d) Has the line \( y = 1 \) as an asymptote

(e) Looks like \( \uparrow \) on the far left and far right

(f) Passes through the origin

(g) Has exactly one vertical asymptote

(h) Has a \( y \)-intercept \((0, b)\) with \( b \) negative

(i) Has a vertical asymptote where the behavior on both sides of the asymptote is the same

(j) Has an oblique asymptote with negative slope

(k) Passes through the point \((-2, 0)\)
6. Let \( H(x) = \frac{(x^2 - 4)(2x^2 - 5x - 3)}{(x^2 - 3x + 2)(x^2 - 25)} \). Determine the following. (3 points each)

\[
\text{provided } x \neq 2
\]

(a) Domain of \( H \)
\( x \neq 2, 1, 5, -5 \)

(b) \( y \)-intercept on graph of \( H \)
\( (0, \frac{-6}{25}) \)

(c) \( x \)-intercept(s) on graph of \( H \)
\( (-2, 0) \)
\( (-\frac{1}{2}, 0) \)
\( (3, 0) \)

(d) Horizontal asymptote on graph of \( H \)
\( y = 2 \)

(e) The coordinates of the “hole” in the graph of \( H \)
\( (2, \frac{20}{21}) \)
7. Let \( f(x) = 16 - 2^x \). Determine each of the following. If the function does not have a particular feature, write NONE. (3 points each)

(a) Domain of \( f \)
\[ (-\infty, \infty) \]

(b) Range of \( f \)
\[ (-\infty, 16) \]

(c) \( x \)-intercept(s) on graph of \( f \)
\[ (4, 0) \]

(d) \( y \)-intercept on graph of \( f \)
\[ (0, 15) \]

(e) Horizontal asymptote on graph of \( f \)
\[ y = 16 \]

(f) Vertical asymptote(s) on graph of \( f \)
NONE

(g) Is the function \( f \) increasing or decreasing?
DECREASING

(h) Describe using shifts and/or reflections how the graph of \( y = f(x) \) is related to the graph of \( y = 2^x \).
reflect \( y = 2^x \) over the \( x \)-axis and then shift the graph up 16 units.