There are 14 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. All answers should be exact unless specified otherwise. There are 125 points available on this exam.

1. Determine which of the following points satisfy the system of equations given below. Circle the letter corresponding to each point that satisfies the system. (8 points)

(a) (2, 0, 7)   (b) (4, 2, 5)  (c) (13, 20, 14)    (d) (5, 0, -2)

\[3x - 2y + z = 13\]
\[-8x + 3y + 4z = 12\]
\[-5x + y + 5z = 25\]

2. Use only the elimination method to find the point of intersection of the following pair of lines. Eliminate x to find y and then eliminate y to find x. DO NOT USE SUBSTITUTION. (10 points)

\[2x + 5y = 3\]
\[4x - 2y = -5\]

\[-4X - 10y = -6\]
\[4X - 2y = -5\]
\[-12y = 11\]
\[y = \frac{-11}{12}\]

\[4x + 10y = 6\]
\[20x - 10y = -25\]
\[24x = -19\]
\[x = \frac{-19}{24}\]

\[\left(-\frac{19}{24}, \frac{11}{12}\right)\]
3. Use only the substitution method to find the point of intersection of the following pair of lines. DO NOT USE ELIMINATION. (10 points)

\[ \begin{align*}
  x - 2y &= 10 \\
  5x + 6y &= 2
\end{align*} \]

\[ \begin{align*}
  x &= 10 + 2y \\
  5(10 + 2y) + 6y &= 2 \\
  50 + 10y + 6y &= 2 \\
  16y &= -48 \\
  y &= -3 \\
  x &= 10 + 2(-3) = 4 \\
  (4, -3)
\]

4. Use the method of elimination and/or substitution to find the solution of the following system of equations. (10 points)

\[ \begin{align*}
  2x - 3y - 5z &= -12 \\
  x + 5y + 2z &= -1 \\
  3x - 2y - z &= 3
\end{align*} \]

\[ \begin{align*}
  x + 5y + 2z &= -1 \\
  2x + 3y + 5z &= 12 \\
  15x - 10y - 5z &= 15 \\
  62x &= 62 \\
  x &= 1 \\
  7.1 + y &= 5 \\
  y &= -2 \\
  3.1 - 2 - 2 - z &= 3 \\
  3 + 4 - z &= 3 \\
  z &= 4
\]

\[ \begin{align*}
  7x + y &= 5 \\
  13x - 7y &= 27 \\
  49x + 7y &= 35 \\
  62x &= 62 \\
  x &= 1 \\
  7.1 + y &= 5 \\
  y &= -2 \\
  3.1 - 2 - 2 - z &= 3 \\
  3 + 4 - z &= 3 \\
  z &= 4
\]
6. For each of the following systems of equations, determine the augmented matrix, the reduced row echelon form of the augmented matrix, and the solution of the system. If the system has no solution write NONE for the solution. (9 points each)

<table>
<thead>
<tr>
<th>System</th>
<th>Augmented Matrix</th>
<th>RREF</th>
<th>Solution</th>
</tr>
</thead>
</table>
| \begin{align*}
2x + y - z &= 4 \\
-x + y + 3z &= 1 \\
2x - 5y + 6z &= -2
\end{align*} | \[
\begin{bmatrix}
2 & 1 & -1 & 4 \\ -1 & 1 & 3 & 1 \\ 2 & -5 & 6 & -2
\end{bmatrix}
\] | \[
\begin{bmatrix}
1 & 0 & 0 & \frac{25}{17} \\ 0 & 1 & 0 & \frac{24}{17} \\ 0 & 0 & 1 & \frac{6}{17}
\end{bmatrix}
\] | \(\left(\frac{25}{17}, \frac{24}{17}, \frac{6}{17}\right)\) |
| \begin{align*}
2x - 2y - 3z &= 7 \\
-3x + 3y + 4z &= -9 \\
-x + y + 2z &= -5
\end{align*} | \[
\begin{bmatrix}
2 & -2 & -3 & 7 \\ -3 & 3 & 4 & -9 \\ -1 & 1 & 2 & -5
\end{bmatrix}
\] | \[
\begin{bmatrix}
1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0
\end{bmatrix}
\] | \((-1+y, y, -3)\) |
| \begin{align*}
2x - 4y + 3z &= 2 \\
4x - y + 2z &= -6 \\
2x + 3y - z &= 7
\end{align*} | \[
\begin{bmatrix}
2 & -4 & 3 & 2 \\ 4 & -1 & 2 & -6 \\ 2 & 3 & -1 & 7
\end{bmatrix}
\] | \[
\begin{bmatrix}
1 & 0 & \frac{5}{14} & 0 \\ 0 & 1 & -\frac{4}{7} & 0 \\ 0 & 0 & 0 & 0
\end{bmatrix}
\] | **NONE** |
7. One of the systems in problem 6 has infinitely many solutions. List three specific solutions of this system. (6 points)

Answers will vary

\((1, 2, -3), (3, 4, -3), (5, 6, -3), \ldots\)

\(y = 2 \quad y = 4 \quad y = 6\)

8. (a) What system of equations is represented by the following augmented matrix? (3 pts)

\[
\begin{bmatrix}
1 & 2 & -3 & 3 \\
0 & 1 & 5 & 4 \\
0 & 0 & 1 & -2 \\
\end{bmatrix}
\]

\[
\begin{aligned}
x + 2y - 3z &= 3 \\
y + 5z &= 4 \\
z &= -2
\end{aligned}
\]

(b) Use back substitution to solve the system of equations in part (a). (5 points)

\[
\begin{aligned}
y + 5z &= 4 \quad \Rightarrow \quad y = 14 \\
y - 10 &= 4 \quad \Rightarrow \quad y = 14 \\
x + 2y - 3z &= 3 \quad \Rightarrow \quad x + 14 - 3(14) = 3 \\
&= -31 \\
x &= 14 - 3(14) = 14 - 42 = -28
\end{aligned}
\]

\((-31, 14, -2)\)

9. Let \(B = \begin{bmatrix} 4 & 5 \\ -2 & 6 \end{bmatrix}\). By hand, calculate the determinant of \(B\). (5 points)

\[4 \cdot 6 - 5 \cdot (-2) = 24 + 10 = 34\]

10. Let \(C = \begin{bmatrix} 6 & 2 & -1 \\ 3 & -5 & 2 \\ 7 & -8 & 4 \end{bmatrix}\). Use your calculator to calculate the determinant of \(C\). (5 pts)

\[-31\]
11. Solve the following system of equations using Cramer’s rule. (10 points)

\[ 2x + 3y = 8 \]
\[ x - 4y = -5 \]

\[ D = \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} = -11 \quad D_x = \begin{vmatrix} 8 & 3 \\ -5 & -4 \end{vmatrix} = -17 \quad D_y = \begin{vmatrix} 2 & 8 \\ 1 & -5 \end{vmatrix} = -18 \]

Solution: \[ \left( \frac{-17}{-11}, \frac{-18}{-11} \right) = \left( \frac{17}{11}, \frac{18}{11} \right) \]

12. Determine the partial fraction decomposition of \( \frac{x^2 - 6x + 8}{(x+1)(x+2)^2} \). (10 points)

\[ \frac{x^2 - 6x + 8}{(x+1)(x+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} \]

\[ x^2 - 6x + 8 = A(x+2)^2 + B(x+1)(x+2) + C(x+1) \]

\[ 1 = A + B \quad 6 = -4A - 3B - C \quad 8 = 4A + 2B + C \]

\[ -6 = 4A + 3B + C \quad 14 = -B \quad B = -14 \]

\[ A = 15 \quad C = -24 \]

\[ \frac{x^2 - 6x + 8}{(x+1)(x+2)^2} = \frac{15}{x+1} + \frac{-14}{x+2} + \frac{-24}{(x+2)^2} \]
13. Determine the partial fraction decomposition of \( \frac{5x-14}{x^3+4x} \). (10 points)

\[
\frac{5x-14}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}
\]

\[
5x-14 = A(x^2+4) + (Bx+C)x
\]

\[
0 = A + B \\
5 = C \\
-14 = 4A \\
\]

\[
A = \frac{-14}{4} = -\frac{7}{2} \\
B = \frac{7}{2} \\
C = 5
\]

\[
\frac{5x-14}{x^3+4x} = \frac{-\frac{7}{2}}{x} + \frac{\frac{7}{2}x+5}{x^2+4}
\]

14. Use a determinant to calculate the area of the triangle whose vertices are (3, –2), (4, 12) and (2, 10). (6 points)

\[
\begin{vmatrix}
3 & -2 & 1 \\
4 & 12 & 1 \\
2 & 10 & 1 \\
\end{vmatrix} = 26
\]

\[
\text{Area} = \frac{1}{2} \cdot 26 = 13 \text{ square units}
\]