

There are 9 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. All answers should be exact unless specified otherwise.

In problems 1-3 determine the factored form, the domain,  $x$ -intercept(s),  $y$ -intercept, vertical asymptotes, and horizontal asymptote on the graph of each of the following rational functions. If a particular feature does not exist, write NONE. (3 points per blank)

1.  $a(x) = \frac{3x+1}{x^2-3x-10}$

Factored Form: \_\_\_\_\_ Domain: \_\_\_\_\_

$x$ -intercept(s): \_\_\_\_\_ Vertical Asymptote(s): \_\_\_\_\_

$y$ -intercept: \_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_

2.  $b(x) = \frac{4x^3+3x^2-6x-5}{5x^2+15x}$

Factored Form: \_\_\_\_\_ Domain: \_\_\_\_\_

$x$ -intercept(s): \_\_\_\_\_ Vertical Asymptote(s): \_\_\_\_\_

$y$ -intercept: \_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_

3.  $c(x) = \frac{x^2 - 4x - 5}{x^2 - 6x + 5}$

Factored Form: \_\_\_\_\_ Domain: \_\_\_\_\_

$x$ -intercept(s): \_\_\_\_\_ Vertical Asymptote(s): \_\_\_\_\_

$y$ -intercept: \_\_\_\_\_ Horizontal Asymptote: \_\_\_\_\_

4. One of the rational functions in problems 1-3 has an oblique asymptote. Identify this function by name and give the slope of the oblique asymptote. (4 points)

5. Determine the least degree polynomial function with real coefficients satisfying the following conditions. Write your answer as a product of factors with real coefficients. (5 points each)

(a) The real zeros of the polynomial are 5 with multiplicity one and  $-2$  with multiplicity two.

(b) The real zeros of the polynomial are 8 with multiplicity three and  $-3$  with multiplicity three. The polynomial also has  $-1 + i$  as a non-real zero.

6. (a) What is the degree of the polynomial in problem 5a? (2 points)

(b) Which of the following describes the left- and right-hand behavior of the graph of the polynomial in problem 5a? Circle the number of the correct response. (3 points)

1. Rises on the left and rises on the right
2. Rises on the left and falls on the right
3. Falls on the left and rises on the right
4. Falls on the left and falls on the right

7. (a) What is the degree of the polynomial in problem 5b? (2 points)

(b) Which of the following describes the left- and right-hand behavior of the graph of the polynomial in problem 5b? Circle the number of the correct response. (3 points)

1. Rises on the left and rises on the right
2. Rises on the left and falls on the right
3. Falls on the left and rises on the right
4. Falls on the left and falls on the right

8. The number  $-1/3$  is a zero of multiplicity two of the following polynomial function. Use this information to determine all of the zeros of this polynomial function and write the polynomial in complete factored form. (12 points)

$$f(x) = 9x^4 + 60x^3 + 199x^2 + 114x + 18$$

Zeros:

Complete Factored Form:  $f(x) =$

9. Sketch the graph of the following rational function. Be sure to show all asymptotes and intercepts on your graph. (10 points)

$$f(x) = \frac{(x-2)(x+1)^2}{(x-3)(x+2)}$$

