In problems 3-8 use our rules for evaluating limits to determine the given limits. . If a particular limit does not exist write DNE and indicate why the limit does not exist.

3. $\lim_{x\to -2}\left(\frac{4x+3}{x^2-2x}\right) = \frac{-5}{8}$

4. $\lim_{x\to 1^+}\sqrt{4x-4} = 0$

5. $\lim_{x\to 1^-}\sqrt{4x-4}$

DNE

$\sqrt{4x-4}$ is not defined for $x < 1$

6. $\lim_{x\to \frac{\pi}{4}}\sin(x) = \sin\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}$

7. $\lim_{x\to 3}\left(\frac{2x^2-5x-3}{x^2-9}\right)$

$= \lim_{x\to 3}\frac{(x-3)(2x+1)}{(x-3)(x+3)}$

$= \frac{7}{6}$

8. $\lim_{x\to 2}(3x^3-4x^2+5x+14)$

$= 3\cdot 8 - 4\cdot 4 + 5\cdot 2 + 14$

$= 32$

9. Let g(x) = $\begin{cases} 2-x^2 & \text{if } x<0 \\ \dfrac{x-4}{x-2} & \text{if } 0<x<2 \\ x^2-3x+4 & \text{if } x\geq 2 \end{cases}$ **. Evaluate each of the following limits. If a particular limit does not exist write DNE and indicate why the limit does not exist.**

(a) $\lim_{x\to 0^-} g(x)$

$= \lim_{x\to 0^-} (2-x^2) = 2$

(b) $\lim_{x\to 0^+} g(x)$

$= \lim_{x\to 0^+} \frac{x-4}{x-2} = 2$

(c) $\lim_{x\to 0} g(x) = 2$

(d) $\lim_{x\to 1} g(x) = \lim_{x\to 1} \frac{x-4}{x-2}$

$= 3$

(e) $\lim_{x\to 2^-} g(x)$

$= \lim_{x\to 2^-} \frac{x-4}{x-2} = \infty$

(f) $\lim_{x\to 2^+} g(x)$

$= \lim_{x\to 2^+} (x^2-3x+4)$

$= 2$

10. (a) Complete the following table of values for the function f(x) = $\frac{\sin(3x)}{x}$.

Round all answers to four decimal places.

x	f(x)
0.5	1.9950
0.1	2.9552
0.01	2.9996
0.001	3.0000
-0.1	2.9552
-0.01	2.9996

(b) Based on your answers to part (a), what do you believe is $\lim_{x\to 0}\left(\frac{\sin(3x)}{x}\right)$?

3

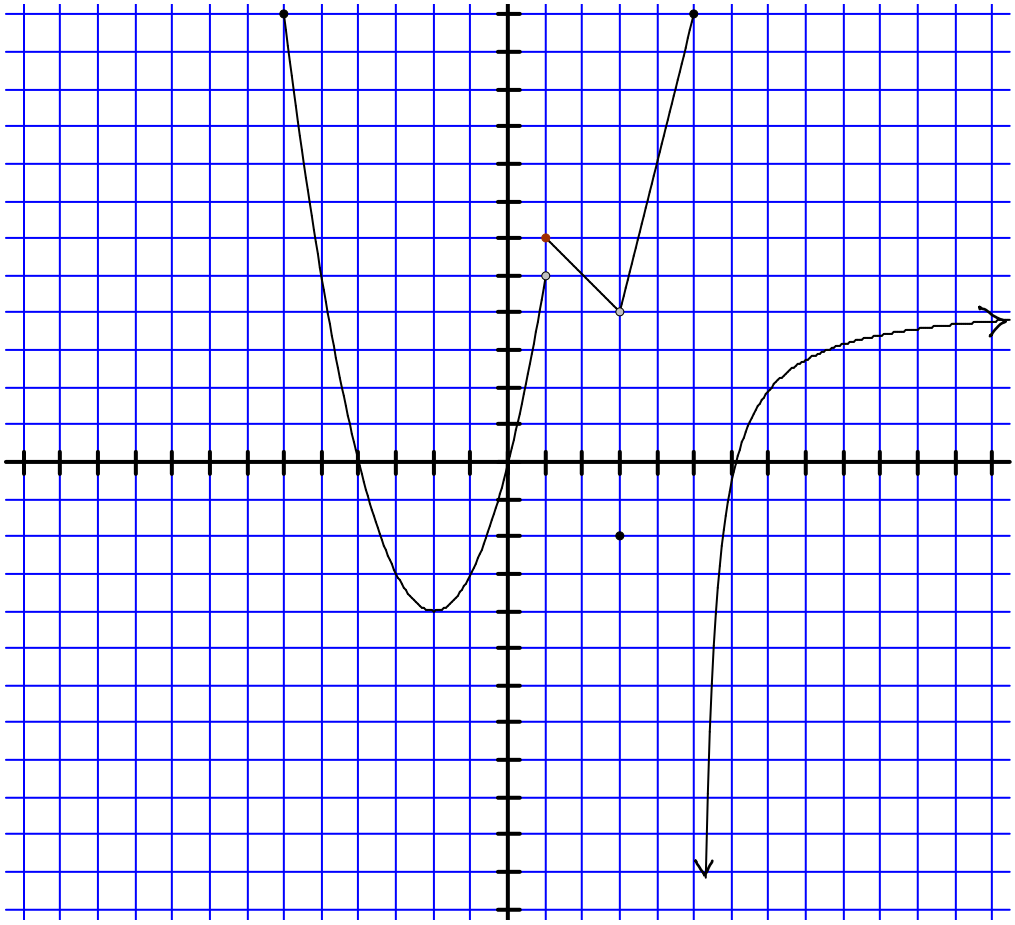
13. Let f(x) = $x^2 - 5x$ and let $a = 3$. For each of the following values of x determine the slope of the secant line through the points (a, f(a)) and (x, f(x)). Round your answers to four decimal places.

x	Slope of Secant Line
2.9	0.9
2.99	0.99
3.1	1.1
3.01	1.01

14. Based on your answers to problem 13 what do you believe is the slope of the tangent line to the graph of y = $x^2 - 5x$ at the point (3, –6)?

1

15. Use the graph of the function shown below to determine each of the following limits. If a particular limit does not exist write DNE and indicate why the limit does not exist.



(a) $\lim_{x\to 1^-} f(x)$

5

(b) $\lim_{x\to 1^+} f(x)$

6

(c) $\lim_{x\to 1} f(x)$

DNE
left & right hand limits are not equal

(d) $\lim_{x\to 3^-} f(x)$

4

(e) $\lim_{x\to 3^+} f(x)$

4

(f) $\lim_{x\to 3} f(x)$

4

(g) $\lim_{x\to 5^-} f(x)$

12

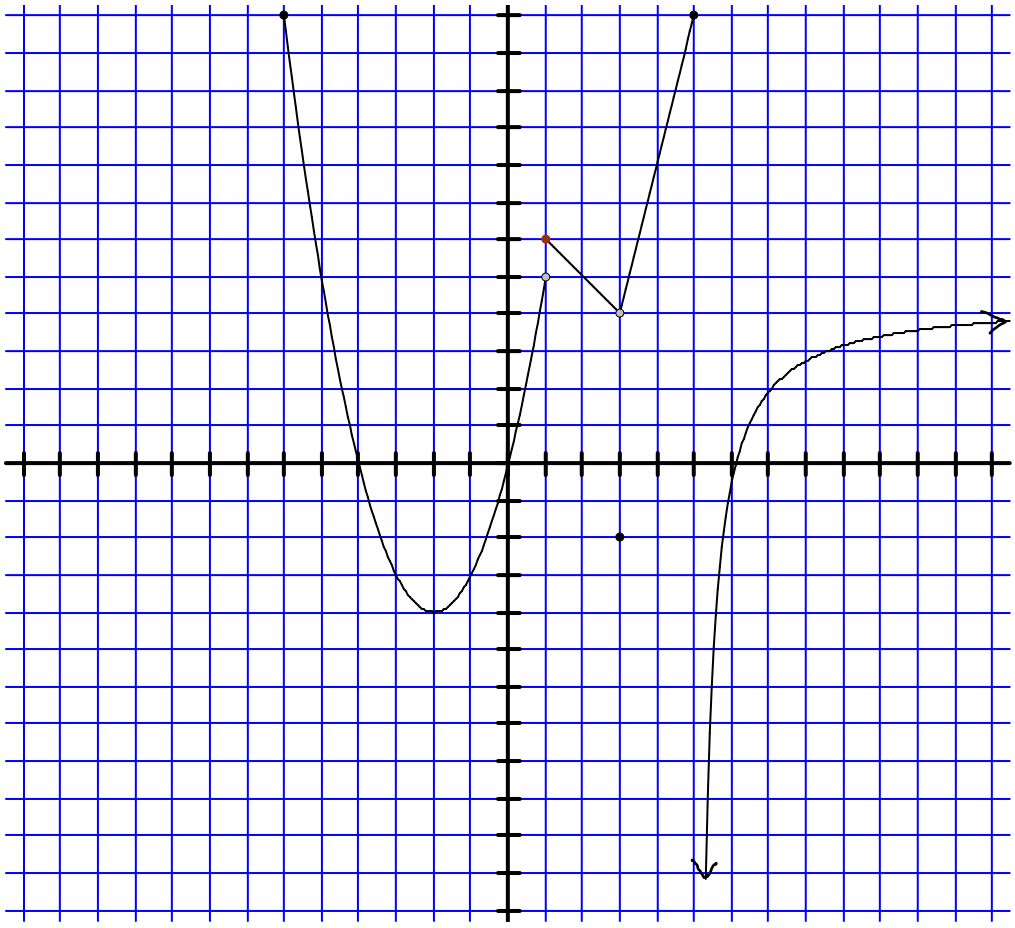
(h) $\lim_{x\to 5^+} f(x)$

$-\infty$

(i) $\lim_{x\to \infty} f(x)$

4

16. Determine the intervals of continuity for the function whose graph is shown below.



$[-6, 1) \cup [1, 3) \cup (3, 5] \cup (5, \infty)$

17. Which discontinuity for the function in problem 16 is removable and how can it be removed?

at $x = 3$ Redefine the function value at 3 to be 4

18. Determine the intervals of continuity for each of the following functions.

(a) $\mathbf{f(x)} = \dfrac{3x^2-5x-2}{x^2+3x-10} = \dfrac{(x-2)(3x+1)}{(x-2)(x+5)}$

$(-\infty, -5) \cup (-5, 2) \cup (2, \infty)$

(b) $\mathbf{g(x)} = \sqrt{x^2-9}$

$(-\infty, -3] \cup [3, \infty)$

(c) $\mathbf{h(x)} = \begin{cases} 3x-5 & \text{if } x<1 \\ x-3 & \text{if } 1<x\le 4 \\ x+2 & \text{if } x>4 \end{cases}$

$(-\infty, 1) \cup (1, 4] \cup (4, \infty)$

19. Two of the functions in problem 18 have a removable discontinuity. Identify which two and indicate how to remove the discontinuity.

f at $x=2$ define $f(2)=1$

h at $x=1$ define $h(1)=-2$

20. Use the limit $\lim_{x\to a}\left(\frac{f(x)-f(a)}{x-a}\right)$ to calculate the slope of the tangent line to the graph of each of the following functions for the specified value of a. Show all necessary work needed to calculate the limit. (5 points each)

(a) f(x) = $\sqrt{3x+1}$ a = 1

$$\lim_{x\to 1}\frac{\sqrt{3x+1}-2}{x-1}=\lim_{x\to 1}\frac{(\sqrt{3x+1}-2)(\sqrt{3x+1}+2)}{(x-1)(\sqrt{3x+1}+2)}$$

$$=\lim_{x\to 1}\frac{3x+1-4}{(x-1)(\sqrt{3x+1}+2)}=\lim_{x\to 1}\frac{3}{\sqrt{3x+1}+2}$$

$$=\frac{3}{4}$$

(b) f(x) = sin(3x) a = 0

$$\lim_{x\to 0}\frac{\sin(3x)-0}{x-0}=\lim_{x\to 0}3\left(\frac{\sin 3x}{3x}\right)=3$$

21. Find the equation of the tangent line to the graph of each of the functions in problem 20 at the point (a, f(a)).

(a) $y-2=\frac{3}{4}(x-1)$

(b) $y-0=3(x-0)$

22. Determine the horizontal asymptotes to the graph of each of the following functions. Indicate whether the asymptote occurs on the left-hand and/or right-hand side of the graph of the function. Show appropriate limit calculations to support your answers.

(a) $y = \dfrac{4x^2+3x-3}{3x^2+4x-1}$ $\quad y = \frac{4}{3}$ on left/right

$$\lim_{x\to\infty} \frac{4x^2+3x-3}{3x^2+4x-1} = \lim_{x\to\infty} \frac{4+3/x-3/x^2}{3+4/x-1/x^2} = \frac{4}{3}$$

Similarly $\lim_{x\to-\infty} \frac{4x^2+3x-3}{3x^2+4x-1} = \frac{4}{3}$

(b) $y = \dfrac{\sqrt{4x^2-x}}{2-x}$ $\quad y=-2$ on right $\quad y=2$ on left

$$\lim_{x\to\infty} \frac{\sqrt{4x^2-x}}{2-x} = \lim_{x\to\infty} \frac{\sqrt{4-1/x}}{2/x-1} = \frac{\sqrt{4}}{-1} = -2$$

$$\lim_{x\to-\infty} \frac{\sqrt{4x^2-x}}{2-x} = \lim_{x\to-\infty} \frac{-\sqrt{4-1/x}}{2/x-1} = \frac{-\sqrt{4}}{-1} = 2$$

23. Consider $\lim_{x\to 2}(3x+5)=11$**.**

(a) For ε = 0.6 determine the best possible δ so that $|3x+5-11|<\varepsilon$ **whenever** $0<|x-2|<\delta$**.**

$$|3x+5-11| = |3x-6| = |3(x-2)|$$

Choose $\delta = 0.2$

(b) Which of the following values of δ will also work when ε = 0.6?

0.60 **0.30** **0.15** **0.05**

(c) For a general value of ε, how should we choose δ?

$\delta \le \varepsilon/3$

24. Consider $\lim_{x \to 2}(x^2) = 4$**.**

For ε = 0.6 determine which of the following values of δ will satisfy $|x^2 - 4| < \varepsilon$ **whenever** $0 < |x - 2| < \delta$**.**

0.60 **0.30** **0.20** **0.15** **0.10** **0.05**