

1. The value of  $\lim_{x \rightarrow 5} \left\{ \frac{3x^2 - 13x - 10}{x^2 - 25} \right\}$  is:

- a) 0
- b) 1
- c) 3
- d) 1.7
- e) not defined

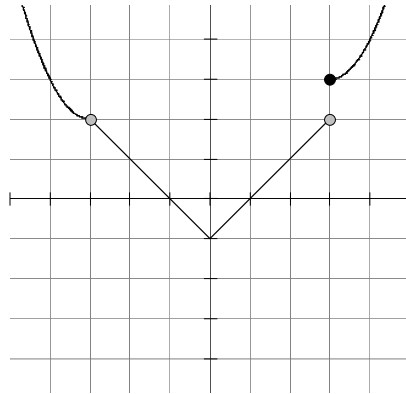
2. The value of  $\lim_{x \rightarrow -\infty} \left( \frac{5 \sin x}{x} \right)$  is:

- a) 1
- b) 5
- c) undefined
- d)  $-\infty$
- e) 0

3. The function  $y = f(x)$  has a nonremovable (jump) discontinuity at  $x = a$  if

- a)  $\lim_{x \rightarrow a} f(x) = 3$  and  $f(a) = 5$
- b)  $\lim_{x \rightarrow a^-} f(x) = 3$  and  $\lim_{x \rightarrow a^+} f(x) = 5$
- c)  $\lim_{x \rightarrow a^-} f(x) = \infty$  and  $\lim_{x \rightarrow a^+} f(x) = -\infty$
- d)  $\lim_{x \rightarrow a^-} f(x) = 3$  and  $\lim_{x \rightarrow a^+} f(x) = \infty$
- e)  $\lim_{x \rightarrow a} f(x) = 3$  and  $f(a) = 3$

Problems 4 and 5 use the following graph of a function:



4. The value of  $\lim_{x \rightarrow 3^+} f(x)$  is:

- a) 3      b) 1      c) 2      d) -1      e) undefined

5. To remove the discontinuity at  $x = -3$ , define a new function  $g(x)$  by:

$$g(x) = \begin{cases} f(x) & \text{if } x \neq -3 \\ k & \text{if } x = -3 \end{cases}$$

What is the value of  $k$ ?

- a) -1      b) 2      c) 0      d) -3      e) The discontinuity cannot be removed.

6. Find the first derivative of  $y = \sec(10x^3 - 11x - 5)$ .

- a)  $(30x^2 - 11) \sec x \tan x$   
 b)  $\sec(10x^3 - 11x - 5) \tan(10x^3 - 11x - 5)$   
 c)  $\sec(30x^2 - 11) \tan(30x^2 - 11)$   
 d)  $(30x^2 - 11) \sec(10x^3 - 11x - 5) \tan(10x^3 - 11x - 5)$   
 e)  $-(30x^2 - 11) \sec(10x^3 - 11x - 5) \tan(10x^3 - 11x - 5)$

7. Find  $\frac{d^{25}y}{dx^{25}}$  if  $y = \cos x$ .

a)  $-\sin x$

b)  $-\cos x$

c)  $\sin x$

d)  $\cos x$

e)  $0$

8. The equation of the line tangent to the graph of  $f(x) = \sqrt{15 - x}$  at the point  $(11, 2)$  is

a)  $y = -\frac{1}{4}x + \frac{19}{4}$

b)  $y = -\frac{1}{4}x + \frac{11}{4}$

c)  $y = -\frac{1}{4}$

d)  $y = \frac{1}{4}x + \frac{11}{4}$

e)  $y = \frac{1}{4}x - \frac{3}{4}$

9. The derivative of  $f(x) = (2x^3 + x + 5)^4$  is:

a)  $4(6x^3 + 1)^3$

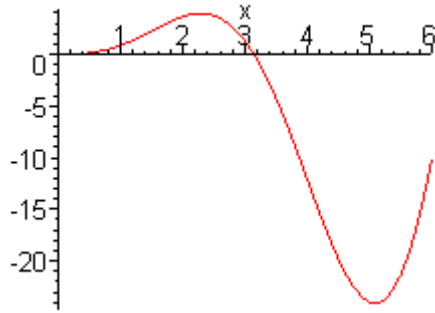
b)  $\frac{1}{5}(6x^3 + 1)^5$

c)  $\frac{1}{5}(2x^3 + x + 5)^5$

d)  $4(2x^3 + x + 5)^3(6x^2 + 1 + 5)$

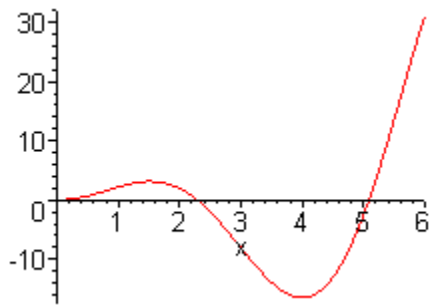
e)  $4(2x^3 + x + 5)^3(6x^2 + 1)$

10. The graph of the first derivative of a function is shown below:

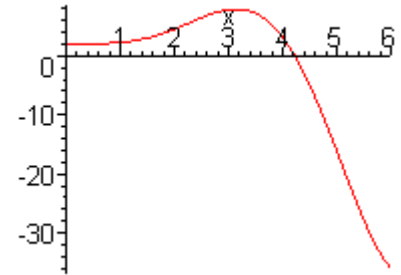


Which of the following graphs could be the graph of the original function?

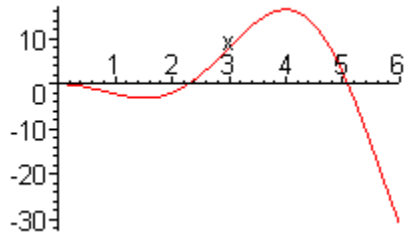
a)



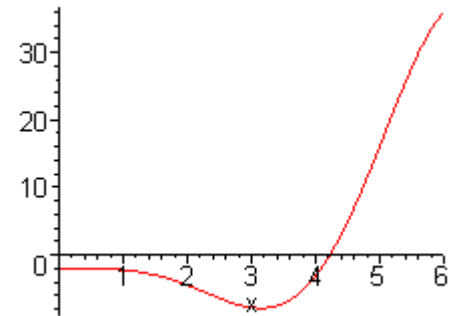
d)



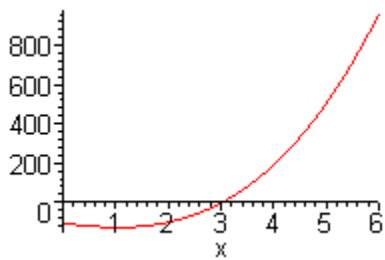
b)



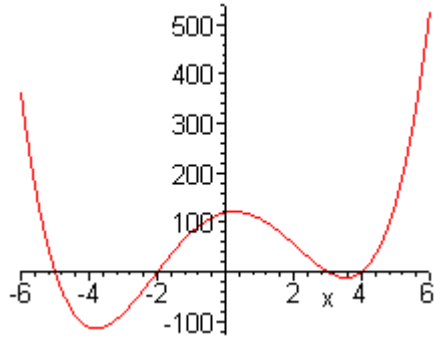
e)



c)

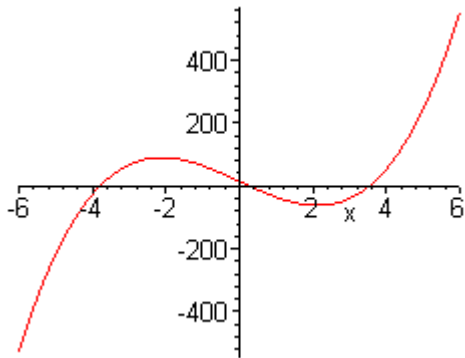


11. The graph of a function is shown below:

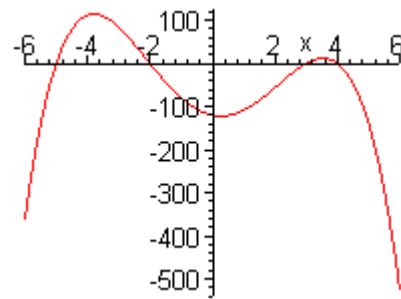


Which of the following graphs could be the graph of the derivative of the function?

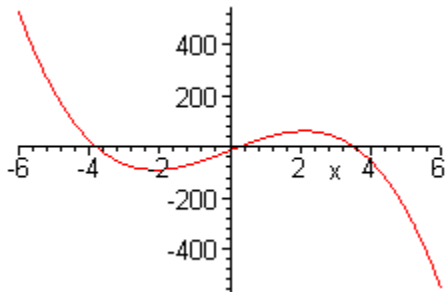
a)



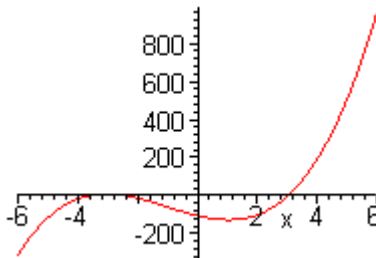
d)



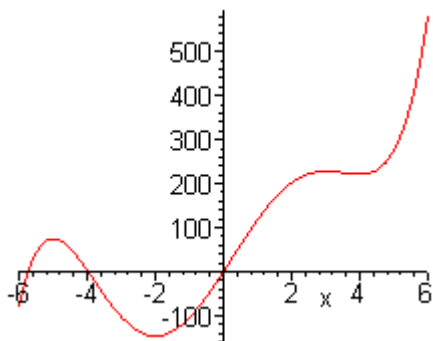
b)



e)



c)



12. Find  $\frac{dy}{dx}$  if  $y^2 + xy + x^2 = 10$ .

a)  $\frac{dy}{dx} = \frac{-3x}{2y}$

b)  $\frac{dy}{dx} = \frac{-2x - 2y}{x}$

c)  $\frac{dy}{dx} = \frac{-2x - y}{2y + x}$

d)  $\frac{dy}{dx} = \frac{-2x}{2y + x}$

e)  $\frac{dy}{dx} = \frac{2x + y}{2y + x}$

13. The absolute maximum value of  $f(x) = (x + 2)^2 (19 - x)$  over the interval  $[-6, 0]$  is:

a) 72

b) 76

c) 400

d) 1372

e) 40572

14. Let  $f(x) = \frac{x}{x^2 + 1}$ . The number of inflection points of the graph of  $f(x)$  is:

a) none

b) one

c) two

d) three

e) four



18. Find  $\frac{d}{dx} \int_2^x \frac{dt}{\sqrt{t^2-1}}$ .

a)  $\frac{1}{\sqrt{x^2-1}}$

b)  $\frac{1}{\sqrt{x^2-1}} - \sqrt{3}$

c)  $\frac{2x}{\sqrt{x^2-1}}$

d)  $\sqrt{x^2-1} - \sqrt{3}$

e)  $\frac{x}{\sqrt{x^2-1}}$

19. Evaluate  $\int \frac{x^4+1}{x^2} dx$ .

a)  $\frac{1}{3}x^3 - \frac{1}{3}x^{-3} + C$

b)  $\frac{1}{3}x^3 - \frac{1}{x} + C$

c)  $\frac{1}{3}x^3 + \frac{1}{x} + C$

d)  $\frac{\frac{1}{5}x^5 + x}{\frac{1}{3}x^3} + C$

e)  $\frac{1}{3}x^3 + x + C$

20. Find  $\int_1^a 12x^2 - 6x + 4 \, dx$ .

a)  $12a^3 - 6a^2 + 4a - 10$

b)  $4a^3 - 3a^2 + 4a - 5$

c)  $12a^3 - 6a^2 - 6$

d)  $4a^3 - 3a^2 - 1$

e)  $4a^3 - 3a^2 + 4a + 5$

21. Find  $\int x(x^2 + 3)^{16} \, dx$ .

a)  $\frac{1}{2}x^2 \left( \frac{x^3}{3} + 3x \right)^{16} + C$

b)  $\frac{1}{34}x^2(x^2 + 3)^{17} + C$

c)  $\frac{x^{49}}{49} + \frac{3x^{17}}{17} + C$

d)  $\frac{1}{34}(x^2 + 3)^{17} + C$

e)  $\frac{1}{17}(x^3 + 3x)^{17} + C$

22. Find  $\int_1^3 \frac{x}{\sqrt{1+3x^2}} dx$ .

a)  $\sqrt{7} - 1$

b)  $\frac{2}{3}(\sqrt{7} - 1)$

c) 0

d)  $\sqrt{7}$

e) Does not exist

23. The velocity of an object at any time  $t$  (in seconds) is given by:  $v(t) = t^2 + \cos t$ . Initially, the object is at position  $s = 10$  feet. What is the object's position (rounded to two decimal places) 8 seconds later?

a) 171.66

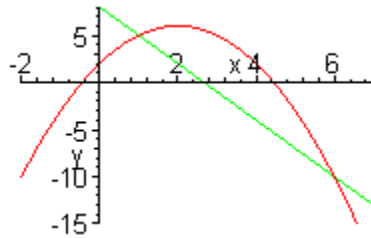
b) 169.68

c) 175.67

d) 179.68

e) 181.66

Problems 24-26 refer to the figure which is the graph of  $y = -x^2 + 4x + 2$  and  $y = -3x + 8$ .



24. The area of the region bounded by  $y = -x^2 + 4x + 2$  and  $y = -3x + 8$  is

a)  $5\frac{1}{6}$  square units

d)  $84\frac{1}{6}$  square units

b)  $9\frac{5}{6}$  square units

e)  $224\frac{1}{6}$  square units

c)  $20\frac{5}{6}$  square units

25. A correct integral for finding the volume of the solid obtained by rotating the region bounded by  $y = -x^2 + 4x + 2$  and  $y = -3x + 8$  about the y-axis using the cylindrical shell method is :

a)  $\int_1^6 2\pi x (-x^2 + x - 6) dx$

b)  $\int_1^6 2\pi x (-x^2 + 7x - 6) dx$

c)  $\int_1^6 \pi (-x^2 + x - 6)^2 dx$

d)  $\int_1^6 \pi (-x^2 + 7x - 6) dx$

e)  $\int_1^6 [\pi (-x^2 + 4x + 2) - \pi (-3x + 8)^2] dx$

26. A correct integral for finding the volume of the solid obtained by rotating the region bounded by  $y = -x^2 + 4x + 2$  and  $y = -3x + 8$  about the line  $y = -13$  is :

a)  $\int_1^6 \pi (-x^2 + x - 19)^2 dx$

b)  $\int_1^6 \pi (-x^2 + 7x + 7)^2 dx$

c)  $\int_1^6 \pi (-x^2 + 7x - 6)^2 dx$

d)  $\int_1^6 [\pi (-x^2 + 4x + 15)^2 - \pi (-3x + 21)^2] dx$

e)  $\int_1^6 [\pi (-x^2 + 4x - 11)^2 - \pi (-3x - 5)^2] dx$

27. Find the average value of the function  $f(x) = 6x^2 - 4x + 1$  on the interval  $[0,5]$ .

a)  $\frac{131}{2}$

b) 75

c) 5

d) 205

e) 41

28. A rectangular tank with base 3 feet by 8 feet and height 12 feet is full of water. Find the work required to pump the water to a point 5 feet above the top of the tank. Assume the density of water is  $62.5 \text{ lb/ft}^3$ .

a) 8250 ft-lbs

b) 198000 ft-lbs

c) 108000 ft-lbs

d) 89250 ft-lbs

e) 216750 ft-lbs

29. Find the derivative of  $f(x) = \sinh x \tanh x$ .

a)  $f'(x) = \sinh x \operatorname{sech}^2 x + \tanh x \cosh^2 x$

b)  $f'(x) = \sinh x \operatorname{sech}^2 x + \tanh x$

c)  $f'(x) = \sinh x \operatorname{sech}^2 x + \tanh x \cosh x$

d)  $f'(x) = \sinh x \operatorname{sech} x + \tanh x \cosh x$

e)  $f'(x) = \sinh x \operatorname{sech} x + \tanh x \cosh^2 x$

30. Define  $f(x) = \int_1^x \frac{dt}{t}$  for  $x > 0$ . Which of the following is not true

a)  $\frac{df(x)}{dx} = \frac{1}{x}$

b)  $f(x)$  is an increasing function

c)  $f(x)$  is a one to one function

d)  $f(1) = 0$

e)  $f(x) \geq 0$  for all  $x$

31. Find the area of the surface obtained by rotating the curve  $y = 2x$  over the interval  $[0,1]$  about the  $y$ -axis.

a)  $\frac{6\pi}{5}$

b)  $\frac{\pi}{5}$

c)  $2\pi\sqrt{5}$

d)  $\pi\sqrt{5}$

e)  $\frac{\pi}{\sqrt{5}}$

32. Find the exact values of the numbers  $c$  that satisfy the conclusion of The Mean Value Theorem for the function  $f(x) = x^3 - 2x$  for the interval  $[-2, 2]$ .

- a)  $\pm 2\sqrt{3}$       b)  $\pm \frac{2\sqrt{3}}{3}$       c)  $\frac{2\sqrt{3}}{3}$       d)  $-\frac{2\sqrt{3}}{3}$       e)  $2\sqrt{3}$

33. Find the absolute minimum values of  $y = 10x^2 - 40x + 8$  on the interval  $[0, 3]$ .

- a) -32      b) 2      c) 32      d) 40      e) -40

34. Find the intervals on which the following function  $f$  is increasing:

$$f(x) = x^3 - 192x + 5$$

- a)  $(-\infty, 24) \cup (24, \infty)$       d)  $(-\infty, 8)$   
b)  $(-8, 8)$       e)  $(-8, \infty)$   
c)  $(-\infty, -8) \cup (8, \infty)$

35. Under ideal conditions a certain bacteria population is known to double every three hours. Suppose that there are initially 100 bacteria. What is the size of the population after 15 hours?

- a) 3200 bacteria      d) 1600 bacteria  
b) 3220 bacteria      e) 13700 bacteria  
c) 6400 bacteria

36. Find the points on the curve  $y = 2x^3 + 3x^2 - 36x + 3$  where the tangent line is horizontal.

a)  $(-2, 71)$  and  $(2, -41)$

d)  $(-3, 84)$  and  $(3, -24)$

b)  $(-3, 84)$  and  $(2, -41)$

e)  $(-2, 71)$  and  $(3, -24)$

c)  $(-2, 71)$ ,  $(2, -41)$ ,  $(-3, 84)$ , and  $(3, -24)$

37. Differentiate the function:  $f(t) = \ln(\cos(6t))$ .

a)  $f'(t) = \frac{6}{\cos(6t)}$

d)  $f'(t) = \frac{1}{\cos(6t)}$

b)  $f'(t) = -6 \cot(6t)$

e)  $f'(t) = -6 \tan(6t)$

e)  $f'(t) = 6 \sec(6t)$

38. At noon, ship A is 160 km west of ship B. Ship A is sailing south at 32 km/hr and ship B is sailing north at 30 km/h. How fast is the distance between the ships changing at 4:00 p.m.? Round the result to the nearest thousandth if necessary.

a) 52.099 km/h

d) 52.098 km/h

b) 52.208 km/h

e) 52.096 km/h

c) 52.048 km/h

39. Use Newton's Method with the initial approximation  $x_1 = 10$  to find  $x_4$ , the fourth approximation to the root of the equation:  $x^3 + x^2 - 6 = 0$ .

a)  $x_4 = 6.09191$

d)  $x_4 = 4.32854$

b)  $x_4 = 2.88191$

e)  $x_4 = 1.44663$

c)  $x_4 = 1.39225$

40. Differentiate the function:  $f(t) = 3\sqrt{t} - \frac{9}{\sqrt{t}}$ .

a)  $f'(t) = \frac{5t\sqrt{t} - 9\sqrt{t}}{2}$

d)  $f'(t) = \frac{1}{2\sqrt{t}} - \frac{11}{2t\sqrt{t}}$

b)  $f'(t) = \frac{3\sqrt{t}}{2} + \frac{11}{2t\sqrt{t}}$

e)  $f'(t) = \frac{3}{2\sqrt{t}} + \frac{9}{2t\sqrt{t}}$

c)  $f'(t) = \frac{3}{\sqrt{t}} - \frac{9}{2t\sqrt{t}}$