Math 320, Exam II

Name _________________________

There are 20 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. Unless specified otherwise, all answers should be exact. Each problem is worth 5 points unless indicated otherwise.

Let \( \vec{p}(t) = \langle t^2 - 3t, t^2 + 1, 2t \rangle \) be the position vector of a particle as a function of time \( t \).

Use this function to work problems 1-12.

1. Determine the velocity \( \vec{v}(t) \) and acceleration \( \vec{a}(t) \) vectors.
   \[
   \vec{v}(t) = \langle 2t - 3, 2t, 2 \rangle \\
   \vec{a}(t) = \langle 2, 2, 0 \rangle
   \]

2. Determine the speed of the particle at time \( t \).
   \[
   \sqrt{(2t - 3)^2 + (2t)^2 + 2^2} = \sqrt{8t^2 - 12t + 13}
   \]

3. Determine the speed, velocity and acceleration at time \( t = 1 \). (9 points)
   
   Speed = 3 \\
   Velocity = \langle -1, 2, 2 \rangle \\
   Acceleration = \langle 2, 2, 0 \rangle

4. Determine the unit tangent vector \( \vec{T}(1) \).
   \[
   \vec{T}(1) = \langle -\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle
   \]

5. Determine \( \vec{v}(1) \times \vec{a}(1) \).
   \[
   \begin{vmatrix}
   \hat{i} & \hat{j} & \hat{k} \\
   -1 & 2 & 2 \\
   2 & 2 & 0
   \end{vmatrix} = -4\hat{i} + 4\hat{j} - 6\hat{k}
   \]

6. Use the answers to problems 3 & 5 to determine the curvature at the point \((-2, 2, 2)\). (6 points)
   \[
   \kappa = \frac{\sqrt{(-4)^2 + 4^2 + (-6)^2}}{3^3} = \frac{\sqrt{68}}{27}
   \]
7. Given that the vector $\mathbf{T}'(t) = \left( \frac{8 + 12t}{(13 + 8t^2 - 12t)^{3/2}}, \frac{26 - 12t}{(13 + 8t^2 - 12t)^{3/2}}, \frac{-16t + 12}{(13 + 8t^2 - 12t)^{3/2}} \right)$, determine the principal unit normal vector $\mathbf{N}(1)$.

$\mathbf{T}'(1) = \left\langle \frac{20}{27}, \frac{14}{27}, -\frac{4}{27} \right\rangle \quad \left| \mathbf{T}'(1) \right| = \frac{\sqrt{612}}{27}$

$\mathbf{N}(1) = \left\langle \frac{20}{\sqrt{612}}, \frac{14}{\sqrt{612}}, -\frac{4}{\sqrt{612}} \right\rangle$

8. Determine the radius of the osculating circle at the point $(-2, 2, 2)$.

$$\frac{27}{\sqrt{612}}$$

9. Determine the binormal vector $\mathbf{B}(1)$.

$$\mathbf{B}(1) = \frac{1}{\sqrt{5508}} \left\langle -36, 36, -54 \right\rangle$$

10. Determine the equation of the osculating plane at the point $(-2, 2, 2)$.

$$-36(x+2) + 36(y-2) - 54(z-2) = 0$$

11. Determine the equation of the normal plane at the point $(-2, 2, 2)$.

$$-\left[ (x+2) + 2(y-2) + 2(z-2) \right] = 0$$

12. Determine the distance traveled by this particle from time $t = 0$ to time $t = 1$. Round your answer to the nearest hundredth.

$$\int_0^1 \sqrt{8t^2 - 12t + 13} \, dt = 3.10$$
13. Suppose the vector \( \vec{v}(t) = (4-8t, 2, \pi \cos(\pi t)), t \geq 0, \) is the velocity vector for a particle moving along a curve. If the position of the particle at time \( t = 1 \) is \((4, -1, 5)\), determine the position of the particle as a function of \( t \).

\[
\vec{p}(t) = \langle 4t-4t^2+c_1, 2t+c_2, \sin(\pi t) + c_3 \rangle
\]

\[
\vec{p}(1) = \langle c_1, 2+c_2, c_3 \rangle
\]

\[
\vec{p}(t) = \langle 4t-4t^2+4, 2t-3, \sin(\pi t) + 5 \rangle
\]

14. At what time will the particle in problem 13 reach the point \((-11, 2, 6)\)?

\[
t = \frac{5}{2}
\]

15. Determine the maximum height above the xy-plane that the particle reaches.

6 units

16. Determine the distance traveled by this particle from time \( t = 1 \) to time \( t = 2 \). Round your answer to the nearest hundredth.

\[
\int_{1}^{2} \sqrt{(4-8t)^2 + 2^2 + (\pi \cos(\pi t))^2} \, dt
\]

\[
\approx 8.59 \text{ units}
\]
17. Consider the surface given by \( \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{9} = 1 \).

(a) What are the restrictions on each of the variables for this surface? (6 points)

For x: \(-2 \leq x \leq 2\)

For y: \(-3 \leq y \leq 3\)

For z: \(-3 \leq z \leq 3\)

(b) If we set z equal to one of its possible values from part (a), the resulting level curves (traces) will be what geometric shape? (3 points)

Ellipse

(c) If we set x equal to one of its possible values from part (a), the resulting level curves (traces) will be what geometric shape? (3 points)

Circles

(d) Which of the following is the name of this surface? Circle the correct response. (3 pts)

- Hyperboloid of one sheet
- Elliptic Paraboloid
- Hyperboloid of two sheets
- Hyperbolic Paraboloid
- Ellipsoid
- Cone
- Elliptic Cylinder
- Hyperbolic Cylinder
- Parabolic Cylinder