Review for Math 320, Exam III

There are more problems on this review than I can ask on a 50-minute exam. I will choose enough problems to get a 100-point exam. Each problem is worth 6 points unless indicated otherwise.

Use the function $f(x, y) = xy(\cos y^2) + 2x^3y^2 - 4y$ in problems 1-5.

1. Determine $\frac{\partial f}{\partial x}$.

2. Determine $\frac{\partial f}{\partial y}$.

3. Determine $\frac{\partial^2 f}{\partial x \partial y}$.

4. Determine the equation of the tangent line to the curve that is the intersection of the surface $z = xy(\cos y^2) + 2x^3y^2 - 4y$ with the plane $x = 3$ at the point $(3, 0, 0)$.

5. Determine $\lim_{(x, y) \to (2, \sqrt{3})} f(x, y)$.
6. Sketch the graph of the domain of the function \( G(x, y) = \sqrt{2 - y - 3x} \).

Let \( F(x, y) = 2x^2 + 2xy + y^2 + 2y \). Use this function for problems 7-14.

7. Determine the directional derivative of \( F \) in the direction of the vector \( \langle 3, -4 \rangle \) at the point \((2, 5)\).

8. In what direction should we go from the point \((2, 5)\) if we want the maximal increase in \( z = F(x, y) \)?

9. Determine the equation of the tangent plane to the surface \( z = F(x, y) \) at the point \((2, 5, 63)\).

10. Determine the equation of the normal line to the surface \( z = F(x, y) \) at the point \((2, 5, 63)\).
11. Determine and identify all local extreme points for $F(x, y) = 2x^2 + 2xy + y^2 + 2y$.

12. Determine the maximum and minimum values of $F(x, y)$ if $(x, y)$ is a point on or inside the triangle with vertices $(4, 0)$, $(0, 6)$, and $(4, 6)$. (18 points)
13. Use differentials to approximate the value of $F(2.02, 4.95)$.

14. Use the chain rule to determine $\frac{\partial F}{\partial r}$ and $\frac{\partial F}{\partial s}$ if $x = 2r^2s^3$ and $y = \cos(rs)$. (10 points)
15. Use Lagrange multipliers to determine the maximum and minimum values of
\( g(x, y) = xy \) subject to the constraint \( x^2 + y^2 = 4 \). (18 points)