There are 10 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. Unless specified otherwise, all answers should be exact.

Use the vector field \( \vec{F}(x, y, z) = \langle 2xz - \sin y, 2y - x \cos y, x^2 - z^2 \rangle \) in problems 1-4.

1. This vector field is conservative. Determine a function \( f \) such that \( \vec{F} = \nabla f \). (6 points)

\[
\begin{align*}
\frac{\partial f}{\partial x} &= 2xz - \sin y & f &= x^2z - x \sin y + g(y, z) \\
\frac{\partial f}{\partial y} &= -x \cos y + \frac{\partial g}{\partial y} & \frac{\partial g}{\partial y} &= 2y & g &= y^2 + h(z) \\
\frac{\partial f}{\partial z} &= x^2 + h'(z) & h'(z) &= -z^2 & h &= -\frac{1}{3}z^3 + c \\
\end{align*}
\]

\[
\nabla f = x^2z - x \sin y + y^2 - \frac{1}{3}z^3 + c
\]

2. Determine \( \text{curl}(\vec{F}) \). (5 points)

\[
\vec{0} = \langle 0, 0, 0 \rangle
\]

3. Determine \( \text{div}(\vec{F}) \). (5 points)

\[
2z + 2 + x \sin y - 2z \\
2 + x \sin y
\]
4. Evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( C \) is the path made up of the line segment from \((-2, 0, 4)\) to \((3, 1, 5)\) followed by the line segment from \((3, 1, 5)\) to \((5, 0, -2)\). **THINK BEFORE YOU DO ANY UNNECESSARY WORK!** (6 points)

\[
\mathbf{F}(5, 0, -2) - \mathbf{F}(-2, 0, 4) = \left( -50 + \frac{8}{3} + C \right) - \left( 16 - \frac{64}{3} + C \right) = -42
\]

5. Rewrite \( \int_0^7 \int_{x-3}^3 3y \, dy \, dx + \int_{-13}^3 \int_{-3y+4}^3 3y \, dy \, dx \) as a double integral in terms of \( u \) and \( v \) using the transformation given by \( u = 3x + 3y - 24 \) and \( v = -x + 3y + 4 \). (15 points)

\[
\int_0^{24} \int_0^{-1} \frac{1}{3} (u+8) \cdot \frac{4}{48} \, dv \, du
\]
Use the vector field \( \vec{G} = <2xy, 2yz, 2xz> \) for problems 6-7.

6. Show that \( \vec{G} \) is not conservative. (5 points)

\[
\text{curl } \vec{G} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
2xy & 2yz & 2xz
\end{vmatrix} = <-2y, -2z, -2x> \neq \vec{0}
\]

7. Evaluate the line integral \( \int_C \vec{G} \cdot d\vec{r} \) where \( C \) is the line segment from (2, 3, 6) to (2, 7, 6). (8 points)

\[
C : <2,3,6> + t <0,4,0> \quad 0 \leq t \leq 1
\]

\[
\int_0^1 <2xy, 2yz, 2xz> \cdot <0,4,0> \, dt = \int_0^1 8(3+4t) \cdot 6 \, dt = 144t + 96t^2 \bigg|_0^1 = 240
\]
8. Use Green’s Theorem to evaluate the line integral \( \int_C \mathbf{H} \cdot d\mathbf{r} \) where \( \mathbf{H} = \langle x^2 - 2y, 3x \rangle \) and \( C \) is the circle of radius 4 centered at the origin. (6 points)

\[
\iint_S 3 - (-2) \, dA = 5 \cdot (16 \pi) = 80\pi
\]

9. On the grid below, draw the vectors associated with the vector field \( \langle y, \frac{1}{x} \rangle \) at the points on the coordinate axes for circles of radius 1 and 2 centered at the origin. (8 points)
10. We want to integrate \( \iiint_R xz \, dV \) where \( R \) is defined to be the set of all points with \( z \geq 0 \) that are inside the cone \( z^2 = x^2 + y^2 \) and inside the sphere \( x^2 + y^2 + z^2 = 18 \).

(a) Set up this integral if the differentials are ordered as \( dz \, dy \, dx \). (8 points)

\[
\begin{align*}
&\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{18-x^2-y^2}} \int_{-\sqrt{9-x^2}}^{x/2} xz \, dz \, dy \, dx \\
&\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{18-x^2-y^2}} \int_{-\sqrt{9-x^2}}^{x/2} xz \, dz \, dy \, dx
\end{align*}
\]

(b) Set up this integral if the differentials are ordered as \( dx \, dy \, dz \). This will require two triple integrals. (8 points)

\[
\begin{align*}
&\int_{0}^{3} \int_{0}^{\sqrt{9-z^2}} \int_{-\sqrt{9-z^2}}^{x/2} xz \, dx \, dy \, dz + \\
&\int_{0}^{3} \int_{-\sqrt{9-z^2}}^{0} \int_{-\sqrt{9-z^2}}^{x/2} xz \, dx \, dy \, dz
\end{align*}
\]

(c) Set up this integral using cylindrical coordinates. (8 points)

\[
\begin{align*}
&\int_{0}^{2\pi} \int_{0}^{3} \int_{0}^{\sqrt{18-r^2}} \rho \cos \theta \cdot \rho \, dz \, d\rho \, d\theta \\
&\int_{0}^{2\pi} \int_{0}^{3} \int_{0}^{\sqrt{18-r^2}} \rho \cos \theta \cdot \rho \, dz \, d\rho \, d\theta
\end{align*}
\]

(d) Set up this integral using spherical coordinates. (8 points)

\[
\begin{align*}
&\int_{0}^{\frac{\pi}{3}} \int_{0}^{2\pi} \int_{0}^{\frac{\sqrt{18}}{\rho}} \rho \sin \phi \cdot \rho \cdot \rho \sin \phi \, d\rho \, d\phi \, d\theta \\
&\int_{0}^{\frac{\pi}{4}} \int_{0}^{2\pi} \int_{0}^{\frac{\sqrt{18}}{\rho}} \rho \sin \phi \cdot \rho \cdot \rho \sin \phi \, d\rho \, d\phi \, d\theta
\end{align*}
\]