Math 140. Test 1 (practice).

Note: This practice test contains problems from tests 1 and 3 of Fall 2002 and test 2 of Spring 2003.

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1. What is the domain of $f(x)$?

$$f(x) = \sqrt{x^2+3}$$

2. What is the domain of $g(x)$?

$$g(x) = \frac{1}{x^2+4x+3}$$

3. What is the domain of $h(x)$?

$$h(x) = \frac{\sqrt{x+2}}{x+1}$$

4. Graph this function:

$$f(x) = \begin{cases} \sqrt{-x}, & x < 0 \\ \sqrt{x}, & x \geq 0. \end{cases}$$

5. Graph this function:

$$g(x) = \begin{cases} x-1, & x < 1 \\ x^2, & 1 \leq x \leq 2 \\ 1-x, & x > 2. \end{cases}$$

6. Let $f(x) = 2x+1$ and $g(x) = x^2 - 1$. Find $f \circ g(x)$ and $g \circ f(x)$.

7. Find the inverse of $y = \sqrt{2x-1}$.

8. Find the inverse of $y = \frac{1+3x}{5-2x}$.

9-12. Find the domain of each function.

9.

$$f(x) = \sqrt{x^2+1}$$

10.

$$f(x) = \frac{1}{x^2-9}$$

11.

$$f(x) = \sqrt{x} + \frac{1}{x-1}$$

12.

$$f(x) = x^3 + 2x^2 + 5x + 1$$

13-15. Graph these piecewise functions:

13.

$$\begin{cases} -1 & x < 0 \\ \sqrt{x} & x \geq 0 \end{cases}$$

14.

$$\begin{cases} -x & x < 0 \\ 2 - x & 0 \leq x \leq 2 \\ 2 & x > 2 \end{cases}$$

15.

$$\begin{cases} \sqrt{-x} & x < 0 \\ 1 & x = 0 \\ \sqrt{x} & x > 0 \end{cases}$$

16, 17. Find the composition $f \circ g(x)$. Give the domain of this composition.

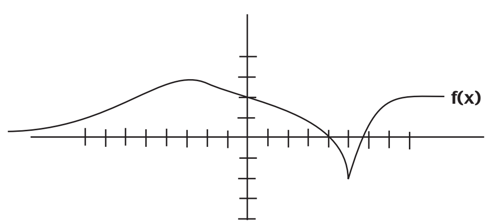
16.

$$f(x) = x^2 + 1 \qquad g(x) = \sqrt{x}$$

17.

$$f(x) = \frac{1}{x-1} \qquad g(x) = 2x$$

18-21. The function $f(x)$ is pictured below. Graph the transformations.



18. $g(x) = 1 - f(x)$

19. $g(x) = f(-x)$

20. $g(x) = 2 + f(x - 3)$

21. $g(x) = -2f(x)$

Solutions.

1. There can't be negative numbers under the square root. So must have $x^2 + 3 \geq 0$. But this is always true. Domain= all real numbers.

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2. Can't have zeros in the denominator of a fraction. So must remove the values of x such that $x^2 + 4x + 3 = 0$. Factor:

$$x^2 + 4x + 3 = 0 \iff (x+1)(x+3) = 0 \iff x = -1 \text{ or } x = -3.$$

Domain= all real numbers except -1 and -3.

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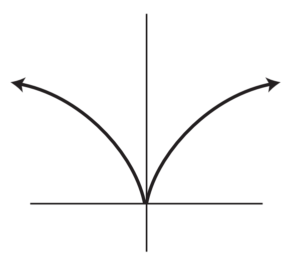
3. Must have both $x + 2 \geq 0$ and $x + 1 \neq 0$. Therefore, $x \geq -2$ and $x \neq -1$. The domain is: $(-2, -1) \cup (-1, \infty)$.

4.



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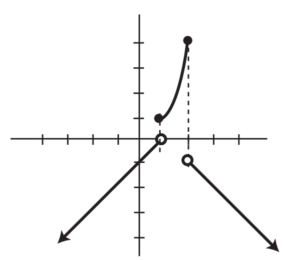


6. Let $f(x) = 2x + 1$ and $g(x) = x^2 - 1$. Find $f \circ g(x)$ and $g \circ f(x)$.

$f \circ g(x) = f(x^2 - 1) = 2x^2 - 1 \quad g \circ f(x) = g(2x+1) = (2x+1)^2 - 1 = 4x^2 + 4x + 1 - 1 = 4x^2 + 4x.$

7. Switch x and y:

$$x = \sqrt{2y-1} \implies x^2 = 2y - 1 \implies x^2 + 1 = 2y \implies y = \frac{x^2+1}{2}$$

Note that it is also necessary to restrict the domain of the inverse function to $x \geq 0$.

8. Switch x and y:

$$x = \frac{1+3y}{5-2y} \implies (5-2y)x = 1 + 3y \implies 5x - 2xy = 1 + 3y \implies 5x - 1 = 3y + 2xy$$

$$\implies 5x - 1 = y(3+2x) \implies y = \frac{5x-1}{3+2x}$$

9. We need $x^2 + 1 \geq 0 \implies x^2 \geq -1$. This is always true since x^2 is always a positive number. The domain is all real numbers.

10. We cannot have $x^2 - 9 = 0$ as this will create a zero in the denominator.

$$x^2 - 9 = 0 \implies x^2 = 9 \implies x = \pm 3$$

The domain is all real numbers except $x = \pm 3$.

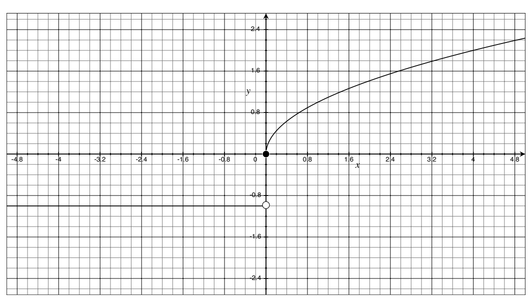
$$D = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$$

11. We must have $x \geq 0$ because of the square root. Also, x cannot be 1 because that would create a zero in the denominator. The domain is then all $x \geq 0$ except $x = 1$:

$$D = [0, 1) \cup (1, \infty)$$

12. The domain is all real numbers.

13.



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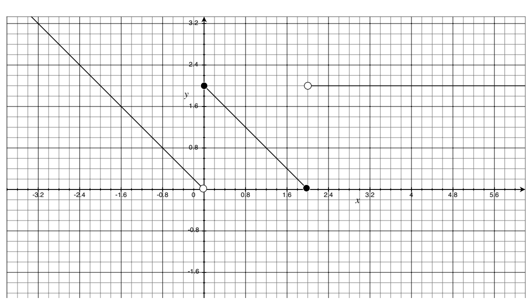

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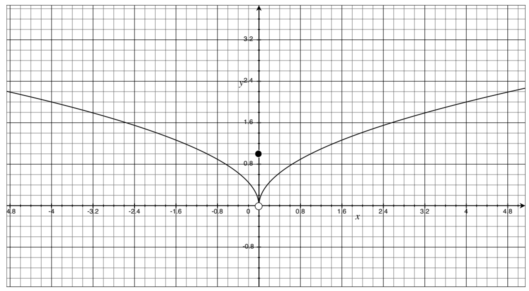


16.

$$f \circ g(x) = f(\sqrt{x}) = (\sqrt{x})^2 + 1 = x + 1$$

Because the domain of $g(x)$ is all $x \geq 0$, and the domain of $f(x)$ is all real numbers, the domain of $f \circ g(x)$ is $[0, \infty)$.

17.

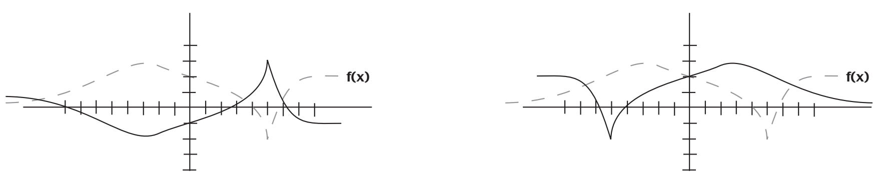
$$f \circ g(x) = f(2x) = \frac{1}{2x - 1}$$

The domain of $g(x)$ is all real numbers. The domain of $f(x)$ is all real numbers except $x = 1$.

$$g(x) = 1 \implies 2x = 1 \implies x = \frac{1}{2}$$

Therefore, the domain of $f \circ g(x)$ is all real numbers except $1/2$: $D = (-\infty, 1/2) \cup (1/2, \infty)$.

18, 19.



20,21.