Math 140. Test 2A. (Harvey Fall 2004)

Name: (5 points) _____

No notes or texts allowed. You may use a TI-83 or TI-86 or equivalent calculator. Show all work. Each problem is worth 10 points.

1. Consider the parabola:

$$p(x) = 2x^2 + 8x + 1$$

Find the coordinates of the vertex. Write the parabola in the standard form $y = a(x - h)^2 + k$.

2. Let

$$p(x) = (x + 1)(2x + 1)^2(-3x + 1)^3(4x - 5)$$

(1) What is the degree? (2) What is the leading coefficient? (3) List each zero and its multiplicity.

3. Let

$$p(x) = x^3 - 11x^2 - 14x + 24$$

Make a list of all *possible* rational zeros of $p(x)$. Find all the real zeros of $p(x)$.

4. Let

$$q(x) = x^3 - 4x^2 - 5x + 20$$

Make a list of all *possible* rational zeros of $q(x)$. Find all the real zeros of $q(x)$.

5. Let

$$p(x) = x^5 + 2x^4 + 3x^3 - 4x - 5$$

According to Descartes' rule of signs, how many positive real zeros can $p(x)$ have? How many negative real zeros can $p(x)$ have?

6. The volume V of a cylinder is jointly proportional to the square of its radius r and its height h. When $r = 1$ and $h = 1$, $V = \pi$. What is the volume of a cylinder with radius $r = 3$ and height $h = 5$?

7. So you think you're fast now? Let's see how fast you are in sixty years! The following data (from 2002) relates record 100m sprint speeds and age. (1) Find the equation of the line that best approximates this data.

age	time
60	11.70
70	12.91
80	14.35
90	18.08
100	43.00

(2) Earlier this year 100 year old 'Flying Phil' Rabinowitz demolished that age group's record, running 100m in 30.86 seconds. Find the equation of the best fit line with this new data.

8. Find the equation of the oblique asymptote of the rational function

$$R(x) = \frac{x^2 + 2x + 5}{x - 3}$$

9. Let

$$R(x) = \frac{x^2 + x - 20}{2x^2 + x - 1}$$

(1) What is the domain of $R(x)$? (2) What are the intercepts of $R(x)$? (3) What are the vertical asymptotes of $R(x)$? (4) What is the horizontal asymptote of $R(x)$? (5) Sketch the graph of $R(x)$, labeling all relevant data.

10. Let

$$R(x) = \frac{x^2 - 2x - 3}{x^2 - 4x + 3}$$

(1) What is the domain of $R(x)$? (2) What are the intercepts of $R(x)$? (3) What are the vertical asymptotes of $R(x)$? (4) What is the horizontal asymptote of $R(x)$? (5) Sketch the graph of $R(x)$, labeling all relevant data.

SOLUTIONS

1.

$$h = -\frac{b}{2a} = -\frac{9}{2 \cdot 2} = -2 \qquad k = p(-2) = 2(-2)^2 + 8(-2) + 1 = -7$$

standard form: $y = 2(x - (-2))^2 + (-7)$

2. The degree is 7. The leading coefficient is $2^2 \cdot (-3)^3 \cdot 4 = -432$.

factor	zero	multiplicity
$x + 1 = 0$	-1	1
$2x + 1 = 0$	-1/2	2
$-3x + 1 = 0$	1/3	3
$4x - 5 = 0$	5/4	1

3. Possible rational zeros are: $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24\}$. Test 1:

$$p(1) = 1 - 11 - 14 + 24 = 0$$

Use synthetic division to divide and get:

$$p(x) = (x-1)(x^2 - 10x - 24)$$

The second parts factors:

$$p(x) = (x-1)(x-12)(x+2)$$

The three zeros are 1, 12, and -2.

4. Possible rational zeros are: $\{\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20\}$. Test 4:

$$q(4) = 4^3 - 4 \cdot 4 - 5 \cdot 4 + 20 = 0$$

Use synthetic division to get:

$$q(x) = (x-4)(x^2 - 5)$$

$$x^2 - 5 = 0 \implies x^2 = 5 \implies x = \pm\sqrt{5}$$

The real zeros of $q(x)$ are 4 and $\pm\sqrt{5}$.

5. $v(p(x)) = 1$ so there is one positive real zero.

$$p(-x) = -x^5 + 2x^4 - 3x^3 + 4x - 5 \implies v(p(-x)) = 4$$

so there are 4, 2, or 0 negative real zeros.

6.

$$V = kr^2h \qquad \pi = k(1)^2(1) \implies k = \pi \implies V = \pi r^2 h$$

When $r = 3$ and $h = 5$:

$$V = \pi(3)^2(5) = 45\pi$$

7. (1) $y = .678x - 34.21$
(2) $y = .435x - 17.21$

8. Long division:

$$\begin{array}{r|l} & x \ +5 \\ \hline x-3 & x^2 + 2x \ +5 \\ & \underline{-x^2 + 3x} \\ & 5x \ +5 \\ & \underline{-5x + 15} \\ & 20 \end{array}$$

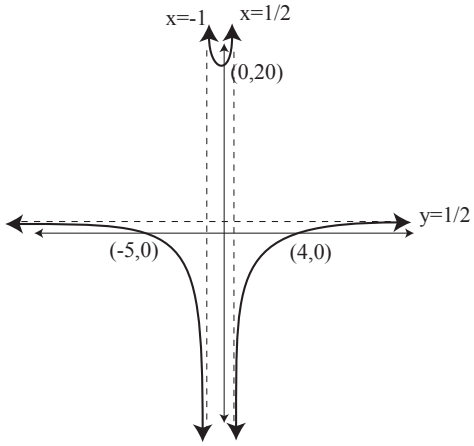
The equation of the oblique asymptote is $y = x + 5$.

9.

$$R(x) = \frac{(x+5)(x-4)}{(2x-1)(x+1)}$$

(1) Domain: all real numbers except -1 and 1/2.
(2) x-int: $(-5, 0)$ and $(4, 0)$. y-int: $(0, 20)$

(3) vertical asymptote: $x = -1$ and $x = 1/2$
(4) horizontal asymptote: $y = 1/2$.
(5)



10.

$$R(x) = \frac{(x+1)(x-3)}{(x-1)(x-3)}$$

(1) Domain: all reals except 1 and 3.
(2) x-int: (-1,0). y-int: (0,-1)
(3) vertical asymptote: $x = 1$
(4) horizontal asymptote: $y = 1$.
(5)