Math 140. Test 3 A (Fall 2004. Harvey).

---

1. Find the solution to this system of equations:

$$\begin{cases} -x + 2y = 20 \\ 3x - y = 10 \end{cases}$$

2. Find all solutions to the system of equations:

$$\begin{cases} x^2 + y^2 - 2y = 0 \\ x^2 + y^2 = 1 \end{cases}$$

3. Graph the solution set of the system of inequalities:

$$\begin{cases} y - 3x \leq 0 \\ y - x \geq 0 \\ x + y \leq 4 \end{cases}$$

Be sure to label all corner points.

4. Consider the following *dependent* system. Write the augmented matrix and use matrix row reduction to find the solution set of this system. Show your work (no calculator).

$$\begin{cases} x + y + z = 5 \\ 2x + y - 3z = 2 \\ -x + y + 9z = 11 \end{cases}$$

5. Find the maximum value of the objective function $z = 6x + y$ on the region satisfying the following inequalities:

$$\begin{cases} x \geq 0 \\ x + y \leq 2 \\ 2x + 4y \geq 5 \end{cases}$$

6. Compute the inverse of the matrix

$$M = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Show your work (no calculator).

7. Perform the matrix arithmetic:

$$\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} + 3 \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix} - 2 \begin{pmatrix} 2 & 2 \\ 3 & -1 \end{pmatrix}$$

8. Perform the matrix multiplication:

$$\begin{pmatrix} x & 1 & 1 \\ 3 & 1 & -1 \\ 0 & 1 & x \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 2 \\ x & 1 & x \\ 1 & 2 & 3 \end{pmatrix}$$

9.

$$M = \begin{pmatrix} 2 & 1 & 1 \\ 4 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$$

What are the minors M_{31}, M_{32} and M_{33}? What are the cofactors A_{31}, A_{32} and A_{33}? What is the determinant of M?

10.

$$M = \begin{pmatrix} 2 & x \\ x & 3 \end{pmatrix}$$

For which values of x is the matrix M *not* invertible?

Solutions.

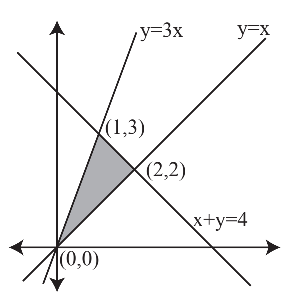
1.

$$\implies y = 3x - 10 \implies -x + 2(3x - 10) = 20 \implies x = 8$$
$$\implies y = 3(8) - 10 = 14 \qquad \text{sol: } (8, 14)$$

2. Subtract the first equation from the second:

$$2y = 1 \implies y = 1/2$$
$$x^2 + y^2 = 1 \implies x = \sqrt{1 - y^2} \implies x = \pm\sqrt{1 - (1/2)^2} = \pm\sqrt{3}/2$$
$$\text{solutions: } \left(\pm\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

3.



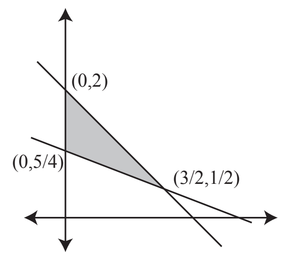
Intersections: $y = 3x$ and $x + y = 4 \implies x = 1,\ y = 3$. $y = x$ and $x + y = 4 \implies x = 2,\ y = 2$.

4.

$$\begin{pmatrix} 1 & 1 & 1 & 5 \\ 2 & 1 & -3 & 2 \\ -1 & 1 & 9 & 11 \end{pmatrix} \xrightarrow[I+III \to III]{-2I+II \to II} \begin{pmatrix} 1 & 1 & 1 & 5 \\ 0 & -1 & -5 & 8 \\ 0 & 2 & 10 & 16 \end{pmatrix} \xrightarrow{2II+III \to III} \begin{pmatrix} 1 & 1 & 1 & 5 \\ 0 & -1 & -5 & 8 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{-II \to II} \begin{pmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{sol: } z = t \qquad y = 8 - 5t \qquad x = 5 - t - (8 - 5t) = -3 + 4t$$

5.



Corners are located at $(0, 2)$, $(0, 5/4)$ and $(3/2, 1/2)$.

$$z(0, 2) = 2 \qquad z(0, 5/4) = 5/4 \qquad z(3/2, 1/2) = 19/2$$

The maximum occurs at $(3/2, 1/2)$.

6.

$$\begin{pmatrix} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{-I+III \to III} \begin{pmatrix} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{pmatrix} \xrightarrow{-II+III \to III} \begin{pmatrix} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{pmatrix}$$

$$\xrightarrow{2III+I \to I} \begin{pmatrix} 1 & 0 & 0 & -1 & -2 & 2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 \end{pmatrix} \xrightarrow{-III \to III} \begin{pmatrix} 1 & 0 & 0 & -1 & -2 & -2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{pmatrix}$$

The inverse is

$$\begin{pmatrix} -1 & -2 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix}$$

7.

$$= \begin{pmatrix} -5 & 1 \\ -5 & 8 \end{pmatrix}$$

8.

$$= \begin{pmatrix} 3x+1 & x+3 & 3x+3 \\ 5+x & 2 & 3+x \\ 2x & 1+2x & 4x \end{pmatrix}$$

9.

$$M_{31} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \qquad M_{32} = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \qquad M_{33} = \begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix}$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1 \qquad A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} = 0 \qquad A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} = -2$$

$$\implies \det(M) = 2 \cdot 1 + 3 \cdot 0 + 1 \cdot (-2) = 0$$

10.

$$\det(M) = 0 \implies 6 - x^2 = 0 \implies x = \pm\sqrt{6}$$