Math 140. Test 3 A (Spring 2004. Harvey).

Name (1 point): ______________________________

No notes or texts allowed. You may use a TI-83 or TI-86 or equivalent calculator. Show all work.

1. (8 points) Use either elimination of substitution to solve the system of equations:

$$\begin{cases} x + 3y = -1 \\ 2x + y = 18 \end{cases}$$

2. (12 points) The system below is a dependent system. Write the augmented matrix for this system, and use matrix row reduction to solve it. What is the solution set?

$$\begin{cases} x - 2y - z = 8 \\ 2x - 3y + z = 23 \\ 4x - 5y + 5z = 53 \end{cases}$$

3. (10 points) How many solutions does the system below have? Show all work.

$$\begin{cases} 2x - 3y - z = 0 \\ -x + 2y + z = 5 \\ 3x - 4y - z = 1 \end{cases}$$

4. (12 points) Graph the solution set of the system of inequalities:

$$\begin{cases} y \leq x^2 \\ y \geq 4x - 3 \end{cases}$$

Be sure to label all corner points.

5. (12 points)

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 4 \end{pmatrix}$$

What are the minors M_{11}, M_{12} and M_{13}? What are the cofactors A_{11}, A_{12} and A_{13}? What is the determinant of M?

6. (14 points) Maximize the function $z = x - y$ subject to the constraints $y \leq 4x$, $x \leq 2$, $y \geq 0$ and $y \leq 1$.

7. (12 points) Compute the inverse of the matrix:

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

8. (10 points) Perform the matrix multiplication:

$$\begin{pmatrix} 2 & 3 & 1 \\ 1 & x & 4 \\ 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 2 \\ 2 & x & 4 \\ 5 & 4 & 3 \end{pmatrix}$$

9. (12 points)

$$\begin{cases} x + y + z = 30 \\ x + 2y + 3z = 68 \\ x + z = 18 \end{cases}$$

Write the corresponding matrix equation. Solve that equation to find the solution to the system (calculator allowed).

SOLUTIONS

1.

$$y = 18 - 2x \implies x + 3(18 - 2x) = -1 \implies -5x + 54 = -1 \implies -5x = -55 \implies x = 11$$
$$y = 18 - 2(11) = -4$$

2.

$$\begin{pmatrix} 1 & -2 & -1 & 8 \\ 2 & -3 & 1 & 23 \\ 4 & -5 & 5 & 53 \end{pmatrix} \xrightarrow[-4I+III \to III]{-2I+II \to II} \begin{pmatrix} 1 & -2 & -1 & 8 \\ 0 & 1 & 3 & 7 \\ 0 & 3 & 9 & 21 \end{pmatrix} \xrightarrow{-II+III \to III} \begin{pmatrix} 1 & -2 & -1 & 8 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The solution:

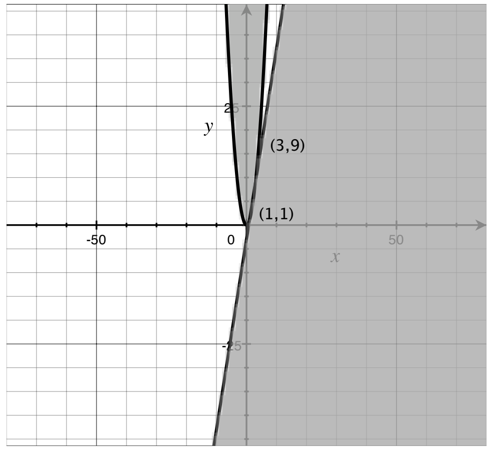
$$z = t \qquad y = 7 - 3t \qquad x = 8 + 2(7 - 3t) + t = 22 - 5t$$

3.

$$I + 2 \times II: \; y + z = 10 \qquad 3 \times II + III: \; 2y + 2z = 16$$
$$\implies \begin{cases} y + z = 10 \\ 2y + 2z = 16 \end{cases} \qquad -2 \times I + II: \; 0 = 8$$

The system is inconsistent: it has no solution.

4. (solution is the unshaded part)



5.

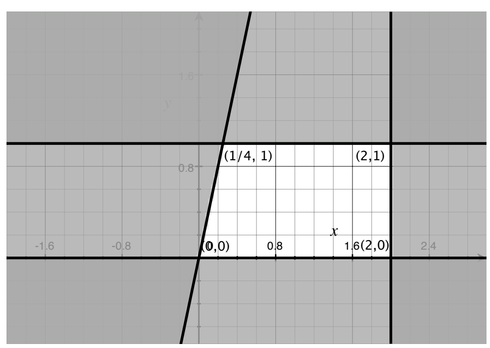
$$M_{11} = \begin{vmatrix} 5 & 6 \\ 2 & 4 \end{vmatrix} = 8 \implies A_{11} = (-1)^{1+1} M_{11} = 8$$

$$M_{12} = \begin{vmatrix} 4 & 6 \\ 1 & 1 \end{vmatrix} = -2 \implies A_{12} = (-1)^{1+2} M_{12} = 2$$

$$M_{13} = \begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix} = 3 \implies A_{13} = (-1)^{1+3} M_{13} = 3$$

$$\det(M) = 8 + 2 + 3 = 13$$

6.



$$z(0,0) = 0 \qquad z(2,0) = 2 \qquad z(2,1) = 1 \qquad z(1/4,1) = 3/4$$

The maximum of 2 occurs at the point $(2, 1)$.

7.

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \xrightarrow[-III+I \to I]{-II+I \to I} \begin{pmatrix} 1 & 0 & 0 & 1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

The inverse is:

$$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

8.

$$= \begin{pmatrix} 13 & 6+3x & 19 \\ 21+2x & 17+x^2 & 14+4x \\ 7 & x+4 & 7 \end{pmatrix}$$

9.

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 30 \\ 68 \\ 18 \end{pmatrix} \implies \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 30 \\ 68 \\ 18 \end{pmatrix} = \begin{pmatrix} 5 \\ 12 \\ 13 \end{pmatrix}$$