

MATH 140. TEST 4 A (FALL 2004. HARVEY).

Name (3 points): _____

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work. Each problem is worth 10 points.

- (5 points)** What is the domain of the function $f(x) = \log_8(x - 3)$? What is the equation of the asymptote?
- (7 points)** Write an equivalent expression that does not contain any products, quotients, powers, or roots:

$$\log\left(\frac{x^2(x-1)^3}{\sqrt{x+1}\sqrt[3]{x+3}}\right)$$

- (7 points)** Write an equivalent expression with only one logarithm:

$$3\log(x-3) + \log(x+1) - \frac{1}{2}\log(x+2)$$

4-6 (7 points each). In problems 4-6, solve for x . Give your answer to two decimal places.

4.

$$e^{3-5x} = 17$$

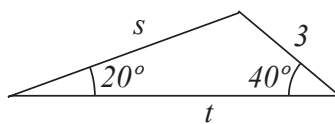
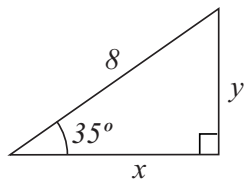
5.

$$\log_5(x+1) - \log_5(x-1) = 2$$

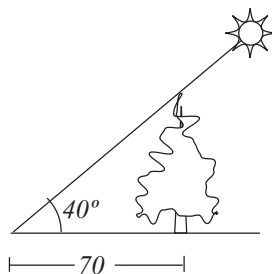
6.

$$e^{2x} - e^x - 6 = 0$$

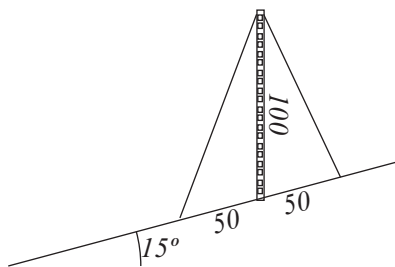
- (10 points)** Suppose \$10,000 is invested into a fund that pay 5% annual interest, compounded continuously. How long will it take for the initial investment to triple?
- (10 points)** A sample taken from Menkaure's Pyramid in Egypt is tested and revealed to have 58% of its original Carbon-14 remaining. From this, estimate how old Menkaure's Pyramid is (recall that the half-life of Carbon-14 is 5730 years).
- (10 points)** Convert the angle measure $23^\circ 13' 45''$ to decimal degrees.
- (12 points)** Find x , y , s , and t :



11. (10 points) We wish to determine the height of a tall tree. We notice that when the sun is 40° above the horizon, the tree casts a 70 foot shadow. How tall is the tree?



12. (10 points) A 100 foot tower is built on a hill with a 15° slope. We wish to add two stabilizing wires from the top of the tower to the ground as illustrated below. How long will the two wires need to be?



SOLUTIONS

1. The domain is $(3, \infty)$. The vertical asymptote is $x = 3$.

2.

$$= 2 \log(x) + 3 \log(x - 1) - \frac{1}{2} \log(x + 1) - \frac{1}{3} \log(x + 3)$$

3.

$$= \log \left(\frac{(x - 3)^3 (x + 1)}{\sqrt{x + 2}} \right)$$

4.

$$\implies 3 - 5x = \ln(17) \implies x = \frac{\ln(17) - 3}{-5} \approx 0.03$$

5.

$$\log_5 \left(\frac{x+1}{x-1} \right) = 2 \implies \frac{x+1}{x-1} = 25$$

$$\implies x+1 = 25(x-1) \implies x+1 = 25x-25 \implies 26 = 24x \implies x = \frac{26}{24} \approx 1.08$$

6.

$$(e^x - 3)(e^x + 2) = 0$$

$$e^x - 3 = 0 \implies e^x = 3 \implies x = \ln 3 \approx 1.10$$

$$e^x + 2 = 0 \implies e^x = -2 \implies \text{no solution}$$

7.

$$30000 = 10000e^{.05t} \implies 3 = e^{.05t} \implies \ln(3) = .05t \implies t = \frac{\ln 3}{.05} \approx 22 \text{ years}$$

8.

$$A = A_0 e^{rt}$$

$$\frac{1}{2}A_0 = A_0 e^{r \cdot 5730} \implies r = \frac{\ln(1/2)}{5730} \approx -1.21 \times 10^{-4}$$

$$.58A_0 = A_0 e^{-1.21 \times 10^{-4} \cdot t} \implies t = \frac{\ln(.58)}{-1.21 \times 10^{-4}} \approx 4500 \text{ years}$$

9.

$$= 23 + \frac{13}{60} + \frac{45}{3600} \approx 23.2292$$

10.

$$x = 8 \cos 35 \approx 6.55 \quad y = 8 \sin 35 \approx 4.59$$

$$\frac{\sin(20)}{3} = \frac{\sin(40)}{s} \implies s = \frac{3 \sin(40)}{\sin(20)} = 5.64$$

$$\frac{\sin(20)}{3} = \frac{\sin(120)}{t} \implies t = \frac{3 \sin(120)}{\sin(20)} = 7.60$$

11.

$$\tan(40) = \frac{x}{70} \implies x = 70 \tan 40 \approx 58.7$$

12. Let x be the length of the longer wire and y be the length of the shorter wire. Use the law of cosines:

$$x^2 = 100^2 + 50^2 - 2 \cdot 50 \cdot 100 \cdot \cos(105) \implies x = 122.8$$

$$y^2 = 100^2 + 50^2 - 2 \cdot 50 \cdot 100 \cdot \cos(75) \implies y = 99.6$$