

MATH 140. TEST 4 (SPRING 2005. HARVEY).

Name (5 points): _____

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

1 (6 points). Give the coordinates of any intercepts and the equations of any asymptotes of each function:

(a) $y = \log_5(x - 2)$

(b) $y = e^{x+3} + 4$

2 (6 points). Write an equivalent expression that does not contain any products, quotients, powers, or roots:

$$\log\left(\frac{x(x+1)^4}{y^3}\right)$$

3 (6 points). Write an equivalent expression with only one logarithm:

$$\frac{1}{2}\log(x) - 2\log(x-1) - 3\log(x+2)$$

4 (8 points). Solve for x :

$$\log_4(x-7) = 3$$

5 (8 points). Solve for x :

$$1 + 2^{3x} = 25$$

6 (10 points). \$10000 is invested into an account earning 3% interest annually, compounded continuously. How long will it take for this initial investment to double?

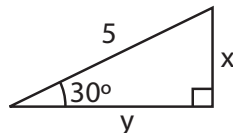
7 (10 points). Archaeologists recently uncovered a tomb in Egypt. Initial samples taken from the tomb were tested and revealed to have 65% of their original Carbon-14 remaining. From this, estimate the age of the tomb (recall that the half-life of Carbon-14 is 5730 years).

8 (6 points). Convert between Degree-Minute-Second notation and Decimal Degree notation:

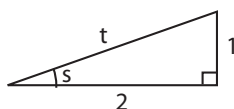
(a) $31^\circ 12' 40'' =$

(b) $21.4175^\circ =$

9 (8 points). Find x and y :

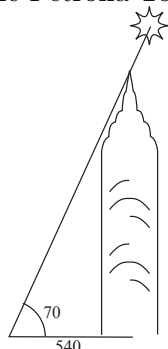


10 (8 points). Find s and t :

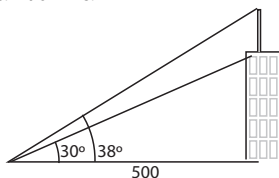


11 (8 points). The Washington Monument is 555 feet high. What is the angle of elevation of the top of the monument from a point on the ground 366 feet away?

12 (8 points). The Petrona Towers in Kuala Lumpur cast a 540 foot shadow when the angle of elevation of the sun is 70° . How tall are the Petrona Towers?



13 (8 points). A radio antenna sits atop a tall building. When viewed from a distance of 500 ft, the angle of elevation to the base of the antenna is 30° and the angle of elevation to the top of the antenna is 38° . How tall is the radio antenna?



SOLUTIONS.

1. (a) vertical asymptote: $x = 2$. intercept $(3, 0)$. (b) horizontal asymptote: $y = 4$, intercept $(0, e^3 + 4) \approx (0, 24)$.

2.

$$= \log(x) + 4 \log(x + 1) - 3 \log(y)$$

3.

$$= \log \left(\frac{\sqrt{x}}{(x-1)^2(x+2)^3} \right)$$

4.

$$\implies x = 7 = 4^3 \implies x - 7 = 64 \implies x = 71$$

5.

$$\implies 2^{3x} = 24 \implies 3x \ln 2 = \ln 24 \implies x = \frac{\ln(24)}{3 \ln 2} \approx 1.528$$

6.

$$20000 = 10000e^{.03t} \implies 2 = e^{.03t} \implies \ln(2) = .03t \implies t = \frac{\ln 2}{.03} = 23.1$$

7.

$$\frac{1}{2}N_0 = N_0e^{5730r} \implies r = \frac{\ln(1/2)}{5730} \approx -1.21 \times 10^{-4}$$

$$\begin{aligned} 0.65N_0 = N_0e^{-1.21 \times 10^{-4}t} &\implies .65 = e^{-1.21 \times 10^{-4}t} \\ &\implies \ln(.65) = -1.21 \times 10^{-4}t \implies t = \frac{\ln(.65)}{-1.21 \times 10^{-4}} \approx 3560 \end{aligned}$$

8. (a) 31.2111 (b) $21^\circ 25' 3''$

9.

$$\sin 30 = \frac{x}{5} \implies x = 5 \sin 30 = 2.5 \quad \cos 30 = \frac{y}{5} \implies y = 5 \cos 30 \approx 4.33$$

10.

$$\tan s = \frac{1}{2} \implies s = \tan^{-1}(1/2) \approx 26.6^\circ \quad t = \sqrt{1+4} = \sqrt{5} \approx 2.24$$

11.

$$\tan \theta = \frac{555}{366} \implies \theta = \tan^{-1}\left(\frac{555}{366}\right) \approx 56.6^\circ$$

12.

$$\tan 70 = \frac{x}{540} \implies x = 540 \tan 70 \approx 1484.$$

13. Let x be the distance from the ground to the top of the antenna. Let y be the height of the building.

$$\tan 38 = \frac{x}{500} \implies x = 500 \tan(38) = 390.6 \quad \tan 30 = \frac{y}{500} \implies y = 500 \tan(30) = 288.7$$

The height of the antenna is $x - y \approx 102$.