

Name (2 points):

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No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

1. (8 points) Consider the parabola defined by the equation:

$$y = 3x^2 + 12x - 5.$$

Find the coordinates of the vertex. Write the parabola in standard form:  $y = a(x - h)^2 + k$ .

2. (9 points) Let

$$p(x) = (3x + 4)^2(x - 20)^3(5x + 3)$$

(1) What is the degree of  $p(x)$ ? (2) What is the leading coefficient of  $p(x)$ ? (3) Describe the limiting behavior of  $p(x)$ .

3. (6 points) What is the *remainder* of the division:

$$\frac{x^4 + 4x^3 + 5x^2 + 3}{x - 1}$$

4. (9 points) Let

$$p(x) = (x - 4)(2x + 3)^3(x + 1)^2$$

(1) List the zeros of  $p(x)$ . (2) Give the multiplicity of each zero. (3) For which zeros will  $p(x)$  cross through the  $x$ -axis and for which zeros will  $p(x)$  touch the axis but not cross through?

5. (10 points) Let

$$p(x) = x^3 + 9x^2 + 19x + 3$$

Make a list of all possible rational zeros of  $p(x)$  (according to the Rational Zeros Theorem). Find all the real zeros of  $p(x)$ .

6. (10 points) Let

$$p(x) = x^4 - 9x^2 - 2x^3 + 12x + 18$$

Make a list of all possible rational zeros of  $p(x)$  (according to the Rational Zeros Theorem). Find all the real zeros of  $p(x)$ .

7. (8 points) Let

$$p(x) = x^7 + x^5 - 3x^4 - x^3 - x^2 + x + 1$$

According to Descartes Rule of Signs, how many positive real zeros may  $p(x)$  have? How many negative real zeros may  $p(x)$  have?

8. (8 points) In origami (paper folding) books, the models are usually arranged from easiest to most difficult. Robert Lang's *Origami Insects and their Kin* is an example. Here are some of the models in the book, their model number, and the number of required steps:

model	model #	# of steps
Treehopper	1	49
Orb Weaver	3	68
Tick	5	69
Butterfly	7	87
Cicada	9	95
Black Pine Sawyer	11	93

Assuming the number of steps increases approximately linearly (a big assumption in this case), how many steps would you estimate are required to fold the Scorpion, model # 20?

**9. (8 points)** The frequency of a vibrating string varies inversely as the length of the string. On a guitar, the length is shortened by pressing down on the fret board. Given a string which is  $650\text{ mm}$  long (standard classical guitar length) and tuned to produce a frequency of  $110\text{ Hz}$  (the note  $A$ ), how far up the string should you press in order to produce a frequency of  $146.8\text{ Hz}$  (the note  $D$ )? Musicians: work it out mathematically!

**10. (12 points)** Let

$$R(x) = \frac{4x^2}{x^2 - 2x - 3}$$

(1) What is the domain of  $R(x)$ ? (2) What are the intercepts of  $R(x)$ ? (3) What are the asymptotes of  $R(x)$ . (4) Sketch the graph of  $R(x)$ , labeling all relevant data.

**11. (12 points)** Let

$$R(x) = \frac{x^3 + 2x^2}{x^2 - 4}$$

(1) What is the domain of  $R(x)$ ? (2) What are the intercepts of  $R(x)$ ? (3) What are the asymptotes of  $R(x)$ . (4) Sketch the graph of  $R(x)$ , labeling all relevant data.

#### SOLUTIONS

1.

$$h = -\frac{b}{2a} = -2 \quad k = 3(-2)^2 + 12(-2) - 5 = -17 \quad y = 3(x + 2)^2 - 17$$

2. The degree is 6. The leading coefficient is 45. The function rises to the left and right.

3.

$$(1)^4 + 4(1)^3 + 5(1)^2 + 3 = 13$$

4.

zero	mult.	behavior
4	1	crosses
$-3/2$	3	crosses
-1	2	touches

5. Possible rational zeros are  $\{\pm 1, \pm 3\}$ . Test to find that  $-3$  is a zero. Divide out by the factor  $x + 3$  to get  $x^2 + 6x + 9$ .

$$x^2 + 6x + 9 = 0 \implies x = \frac{-6 \pm \sqrt{6^2 - 4}}{2} = -3 \pm \sqrt{8}$$

The real zeros are  $-3, -3 \pm \sqrt{8}$ .

6. Possible rational zeros are  $\pm\{1, 2, 3, 6, 9, 10\}$ . Test to find that  $-1$  is a zero, and divide out by the factor  $x + 1$  to get  $x^3 - 3x^2 - 6x + 18$ . Test to find  $3$  is a zero. Divide out by the factor  $x - 3$  to get  $x^2 - 6$ . Solve  $x^2 - 6 = 0 \implies x = \pm\sqrt{6}$ . The zeros are  $-1, 3, \pm\sqrt{6}$ .

7. There are 2 sign changes of  $p(x)$ , hence there are 2 or 0 positive real zeros. There are 3 sign changes of  $p(-x)$ . Hence there are 3 or 1 negative real zeros.

8. The best fit line is:

$$y = 4.56x + 49.49$$

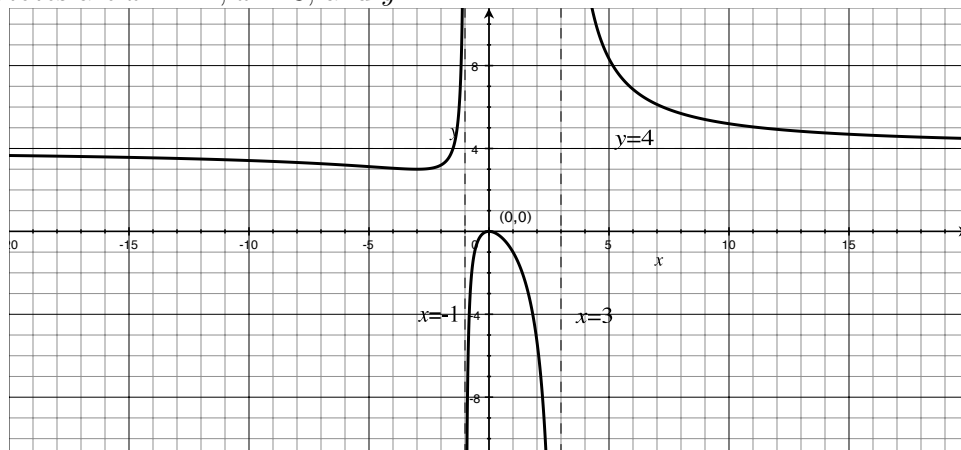
When  $x = 20$ ,  $y \approx 141$  (note: the actual number of steps for this model is 157).

9.

$$110 = \frac{k}{650} \implies k = 71500 \qquad 146.8 = \frac{71500}{L} \implies L = 487.$$

Press  $650-487 = 163$  mm up the neck.

10. The domain is all real numbers except  $3$  and  $-1$ . The  $x$ -intercept is  $0$ . The  $y$ -intercept is  $0$ . The asymptotes are  $x = -1$ ,  $x = 3$ , and  $y = 4$ .



11. The domain is all real numbers except  $\pm 2$ . The  $x$ -intercept is  $0$ . The  $y$ -intercept is  $0$ . There is a vertical asymptote  $x = 2$ . Divide to find the equation of the slant asymptote:  $y = x + 2$ .

